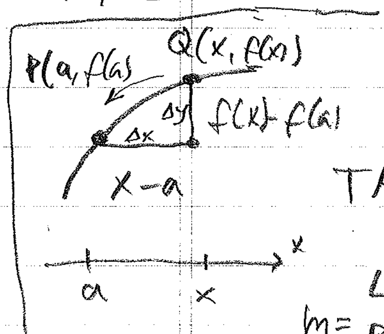
SHEET # 0275 :

DERIVATIVE AT A POINT : 2 DEFINITIONS

SLOPE OF SECANT LINE

FIGURE A



$$m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{f(x_Q) - f(x_P)}{x_Q - x_P} = \frac{f(x) - f(a)}{x - a}$$

$$= \frac{f(a+\Delta x) - f(a)}{\Delta x}$$

TANGENT : $Q \to P$.

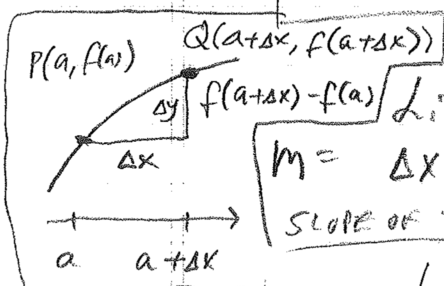
$$m = \lim_{P \to Q} m_{PQ} = \lim_{x_Q \to x_P} \frac{f(x_Q) - f(x_P)}{x_Q - x_P}$$

SLOPE OF TANGENT LINE AT $x = a$

$$\boxed{m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

Definition 1

FIGURE B



$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$$

SLOPE OF TANGENT LINE AT $x = a$

$$\boxed{m = \lim_{H \to 0} \frac{f(a+H) - f(a)}{H}}$$

Definition 2

Recall $(b-c)(b+c) = b^2 - c^2$

EXAMPLE : SLOPE AT $x = a$ FOR $y = f(x) = \sqrt{x}$

DEFINITION 1

$$m_{PQ} = \frac{\sqrt{x} - \sqrt{a}}{x - a} = \text{(multiply by CONJUGATE trick)}$$

$$= \frac{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}{x - a \quad (\sqrt{x} + \sqrt{a})}$$

$$= \frac{x - a}{x - a} \frac{1}{\sqrt{x} + \sqrt{a}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{a}}$$

$$m = \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{\sqrt{a} + \sqrt{a}} = \boxed{\frac{1}{2\sqrt{a}}} \rightarrow \frac{1}{2y}$$

EXERCISE: SLOPE OF $f(x) = x^2$ AT $x = a$ WITH 2 DEFINITIONS.

DEFINITION 2

$$m_{PQ} = \frac{\sqrt{a+H} - \sqrt{a}}{H} = \text{(multiply BY CONJUGATE "trick")}$$

$$= \frac{(\sqrt{a+H} - \sqrt{a})(\sqrt{a+H} + \sqrt{a})}{H \quad (\sqrt{a+H} + \sqrt{a})}$$

$$= \frac{(a+H) - a}{H(\sqrt{a+H} + \sqrt{a})} = \frac{H}{H} \frac{1}{\sqrt{a+H} + \sqrt{a}}$$

$$= \frac{1}{\sqrt{a+H} + \sqrt{a}}$$

$$m = \lim_{H \to 0} \frac{1}{\sqrt{a+H} + \sqrt{a}} = \frac{1}{\sqrt{a} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$$