Calculus

Sheet #500: Integrals, Diff Eqs

Name _____ Per/Sec. _____

1. The following table shows selected coordinates for $y = f(x)$:

x	1	2	3	4
y	1.2	2.3	2.5	4.9

Given that f is continuous on $[1, 4]$, find a trapezoidal approximation, with $n = 3$, for the area under the curve from $x = 1$ to $x = 4$.

2. The following table shows selected coordinates for $y = f(x)$:

x	1	2	3	4	5
y	1.2	3.8	6.4	15.2	26.2

Given that f is continuous on $[1, 5]$, find a trapezoidal approximation, with $n = 4$, for the area under the curve from $x = 1$ to $x = 5$.

3. Integrate: $\int 8\,dx$

4. $\int (4x^3 - 3x^2)\,dx =$.

5. Integrate: $\int (3x^2 - 2x + 5)\,dx$

6. Integrate: $\int \frac{1}{x^3}\,dx$

7. Integrate: $\int \frac{1}{t^6}\,dt$

8. Find $\int u^5\,du$ where $u = x^3 - 5$.

9. Use the Fundamental Theorem of Calculus to evaluate $\int_2^3 x^3\,dx$.

10. Use the Fundamental Theorem of Calculus to evaluate $\int_{-2}^1 (1 - 2x)\,dx$.

11. Use the Fundamental Theorem of Calculus to evaluate $\int_1^9 \sqrt{x}\,dx$.

12. $\int_0^a \sqrt[3]{x}\,dx =$

13. Choose the correct statement given that $\int_0^7 f(x)\,dx = 8$ and $\int_1^7 f(x)\,dx = -3$.

A) $\int_7^1 f(x)\,dx = -3$ B) $\int_0^1 f(x)\,dx = 5$

C) $\int_1^0 f(x)\,dx = 11$ D) $\int_0^1 f(x)\,dx = 11$

E) $\int_0^1 f(x)\,dx = -11$

14. Choose the correct statement given that $\int_0^9 f(x)\,dx = 5$ and $\int_3^9 f(x)\,dx = -1$.

A) $\int_3^0 f(x)\,dx = 6$ B) $\int_9^3 f(x)\,dx = -1$

C) $\int_0^3 f(x)\,dx = 4$ D) $\int_0^3 f(x)\,dx = 6$

E) $\int_0^3 f(x)\,dx = -6$

15. Given $f(x) = Ax^3 + Bx^2 + cx + D$, and

I. $f(1) = 4$
II. $f'(1) = -1$
III. $f''(0) = -10$
IV. $f''(1) = 2$

What is the value of $(A + B + C + D)$?

16. $\int x(x^2 - 1)^4\,dx =$

17. $\int x\sqrt{4 - 9x^2}\,dx =$

18. $\int \cos\frac{x}{3}\,dx =$

19. $\int \frac{\cos(\ln x)}{x}\,dx =$

20. If $\frac{dy}{dx} = \sin^5 x \cos x$, then $y =$

21. $\int \frac{\ln(5x)}{x}dx =$

22. $\int e^{-5x}dx =$

23. Integrate: $\int \sin 3x\,dx$

24. $\int \frac{dx}{25 + x^2} =$

25. $\int \frac{9\,dx}{\sqrt{4 - x^2}} =$

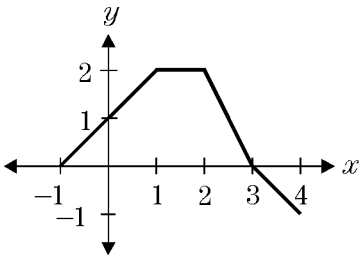
26. On the planet Mars the population of a newly discoverd bacteria in the year 1990 was about 5 billion. If the population was growing according to $P(t) = 5e^{0.017t}$ then which definite integral gives the population for the 8-year period starting from the year 1990. Assume $t = 0$ at the beginning of the year 1990.

A) $\int_0^8 5e^{0.017t}\,dt$ B) $8\int_0^1 5e^{0.017t}\,dt$

C) $\int_0^1 40e^{0.017t}\,dt$ D) $\int_{1990}^{1998} 5e^{0.017t}\,dt$

E) $\int_0^{1998} 5e^{0.017t}\,dt$

27. The graph of f is shown for $-1 \le x \le 4$. What is the value of $\int_{-1}^{4} f(x)\,dx$?



28. If $\int_1^6 f(x)\,dx = 9$ and $\int_1^6 g(x)\,dx = -5$ then what is the value of $\int_1^6 (f-g)(x)\,dx$?

29. If $\int_1^5 f(x)\,dx = 3$ and $\int_1^5 g(x)\,dx = -9$ then what is the value of $\int_1^5 (3f-2g)(x)\,dx$?

30. If $\int_0^a x^3\,dx = k$ for $a > 0$ then, in terms of k, $5\int_0^a x^3\,dx =$ _____

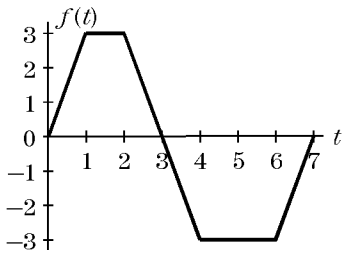
31. If $\int_0^a x^3\,dx = k$ for $a > 0$ then, in terms of k, $\int_{-2}^{2} x^3\,dx =$ _____

32. If $\int_0^a x^3\,dx = k$ for $a > 0$ then, in terms of k, $\int_2^{a+2} (x-2)^3\,dx =$ _____

33. If $\int_0^a x^3\,dx = k$ for $a > 0$ then, in terms of k, $\int_0^a (5+x^3)\,dx =$ _____

34. $\int_{\frac{\pi}{2}}^{x} \sin t\,dt =$?

35.



The graph of $f(t)$ is shown. If $F(x) = \int_0^x f(t)dt$, then fill in the values of $F(x)$ asked for in the table,

x	0	1	2	3	6	7
$F(x)$						

36. Given $\int_{-2}^{7} f(x)dx = 10$ and $\int_{-2}^{0} f(x)dx = -3$ then evaluate

a) $\int_{0}^{7} f(x)dx$

b) $\int_{2}^{11} f(x-4)dx$

37. Given $\int_{-1}^{8} f(x)dx = 7$ and $\int_{-1}^{0} f(x)dx = 4$ then evaluate

a) $\int_{0}^{8} f(x)dx$

b) $\int_{-4}^{-3} f(x+3)dx$

38. A caribou population is being eradicated at a rate of $D(t) = 50e^{-0.2t}$ thousand per year. Approximately how many thousands of caribou will be eradicated in the second year from now?

39. A story is being heard at a rate of $R(t) = 400e^{-0.21t}$ people per week. Approximately how many people will hear the story during the seventh and eighth week?

40. Approximate: $\int_{0}^{6.931} (x^2 - 5)\,dx$

41. Approximate: $\int_{0}^{3/4} \cos(x^2)\,dx$

42. Evaluate: $\int_{0}^{\pi/4} x\cos x\,dx$

43. $\int_{0}^{2} e^{-x^2}\,dx =$

44. Find $\frac{d}{dx}\int_{1}^{x} (5c^4 - 3c + 7)dc =$

45. Evaluate $\frac{d}{dx}\int_{2}^{x} t^{-2}\,dt$ for $x > 1$.

46. $\frac{d}{dx}\int_{5}^{x} \sqrt{2\sin^2 t - 7}\,dt =$

47. $\frac{d}{dx}\int_{x}^{2} \frac{5t}{2t^3 - 3}\,dt =$

48. $\frac{d}{dx}\int_{-2}^{x^4} (t-5)dt =$

49. $\lim_{h\to 0} \frac{1}{h}\int_{x}^{x+h} (-t\sqrt{\cos t})dt =$

50. If $\int_0^5 f(x+k)dx = 3$, then $\int_{5+k}^{k} f(x)dx =$

51. If $F(x) = \int_{2x+3}^{5} f(t)dt$ and $f(17) = 4$, then $F'(7) =$

52. If $F'(x) = f(x)$ for all x, and if f is a continuous function, then $\int_1^4 f(4x)\,dx =$

53. If $F'(x) = f(x)$ for all x, and if f is a continuous function, then $\int_1^6 f(5x)\,dx =$

54. If $F'(x) = f(x)$ for all x, and if f is a continuous function, then $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin(2x)\,dx =$

55. If $\int_k^{\frac{\pi}{6}} \cos x\,dx = 0.3$, then $k =$

56. $\frac{d}{dx}\int_{7x}^{x^4} \sqrt{t^2-1} =$ _____

57. $\frac{d}{dx}\int_{\sin x}^{\cos x} t^3 =$ _____

58. Solve the differential equation $\frac{dy}{y^2} = x\,dx$.

59. Solve the differential equation $e^{x+y}\,dy = dx$.

60. Solve the differential equation $\frac{2}{x}\frac{dy}{dx} = e^{2y}$ given $y(0) = -1$.

61. Find the equation of the family of curves $\frac{dy}{dx} = 6x^2 - 8x$ that passes through the point $(-1, -5)$.

62. Find the solution of the differential equation $\frac{dy}{dx} = x^4 + x$ given that the origin is on the curve.

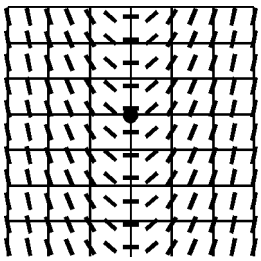
63. Given $\frac{dy}{dx} = \sin x$, find the equation of the family of curves which pass through the point $(\pi, 3)$.

64. Given $\frac{dy}{dx} = e^{5x}$ find the equation of the family of curves which pass through the point $(-\ln 1, 0)$.

65. Solve $yy' = 8xe^{2x^2}$, for $x \geq 1$, $y \geq 1$, and $y(0) = 1$.

66. Which of the following differential equations goes with the slope field shown?

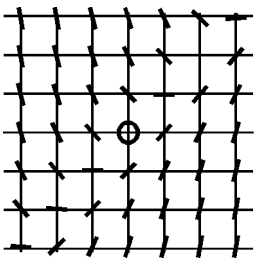
A) $y' = -x$
B) $y' = -x^2$
C) $y' = -2x$
D) $y' = 2x$
E) $y' = x^2$



$[-3, 3] \times [-4, 3]$

67. Which of the following differential equations goes with the slope field shown?

A) $y' = x - y$
B) $y' = x + y$
C) $y' = x^2 + y^2$
D) $y' = \frac{x}{y}$
E) $y' = -\frac{x}{y}$



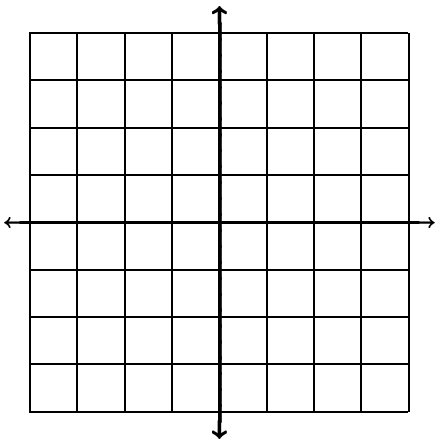
$[-3, 3] \times [-3, 3]$

68. Consider the differential equation: $\frac{dy}{dx} = -2x + 4$.

a) Complete the table for the slopes at the following points:

$x, y \rightarrow$ / $\downarrow$	-1	0	1	2	3	4
-1						
0						
1						
2						
3						
4						

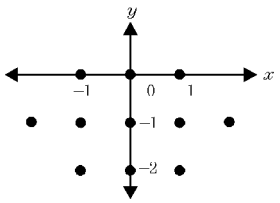
b) Now draw the slope field for the given differential equation on the grid:



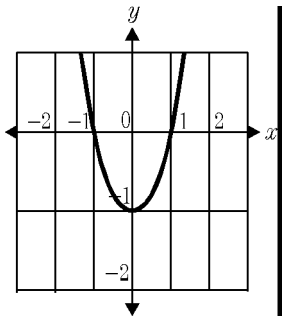
c) Start at the point $(2, 4)$ and sketch the graph by going to the right and left of the point.

d) What does this shape represent?

69. a) Given the differential equation $\frac{dy}{dx} = 2x(y+1)^2$, sketch its slope field at the eleven points indicated.



b) Use the slope field from part (a) to explain why a solution such as the one shown below is impossible.



70. Find the average value of $f(x) = x^2$ over the interval $[2, 5]$

71. Find the average value of $4x$, over the interval $a \le x \le b$.

72. Given $g(x) = A + h(x)$ and $\int_1^5 h(x)\,dx = A$, find the average value of $g(x)$ over the interval $[1, 5]$ in terms of A.

73. A projectile moves on the x-axis so that its position is $x(t) = \frac{1}{2}\sin 2t + \frac{3}{4}$, where $t \ge 0$. For $0 \le t \le \frac{\pi}{2}$, find the average value of the position function.

74. Find the average value of $f(x) = 0.4e^{-2x^2}$ on the closed interval $[-1, 1]$.

75. Find the average value of

$$f(x) = \begin{cases} x+7 & \text{for } -7 \le x < 0, \\ -3x+9 & \text{for } 0 \le x \le 3 \end{cases}$$

on $[-7, 3]$.

76. A bacteria population is estimated to be 20 million and the population h hours from now is estimated to be $P(t) = 20e^{0.029t}$. Approximately what will the average population be over the next day?

77. A function is defined by the following properties:

$f'(x) = 6x$ and $f(1) = -1$. Find $f(x)$.

78. A function is defined by the following properties:

$f''(x) = 12x$; $f'(1) = 1$; and $f(2) = 16$. Find $f(x)$.

79. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 9t^2 + 8t - 6$. Which of the following expressions could represent the position $x(t)$ of the particle at any time $t \geq 0$?

A) $3t^3 + 4t^2 - 6t + 5$ B) $3t^3 + 4t^2 - 6$

C) $9t^3 + 8t^2 - 6t$ D) $\frac{t^3}{3} + 8t^2 - 6t$

E) $9t^2 + 8t - 6$

80. Find the position function, $P(t)$, given $a(t) = 6$, $v(1) = 11$, and $P(2) = 14$. $a(t)$ is the acceleration and $v(t)$ is the velocity.

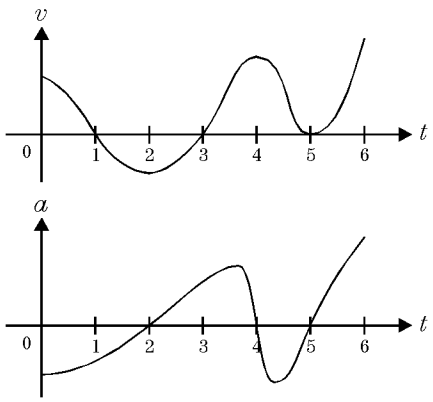
81. An object moves such that its acceleration $a(t) = 6(1 - 2t)$. Given $v(0) = 2$ and $x(0) = 1$, find $x(2)$.

82. Use $a(t) = -10\,\text{ft/s}^2$ as the acceleration due to gravity on the planet Mathematica. A rock is thrown vertically upward from the ground with an initial velocity of 40 feet per second. How high will the rock go?

83. Use $a(t) = -45\,\text{ft/s}^2$ as the acceleration due to gravity. A rock is thrown vertically upward from the ground with an initial velocity of 60 feet per second. For how many seconds will the rock be going upward?

84. A motion is described by $s = f(t)$ for $t \geq 0$, with velocity and acceleration graphs as illustrated.

Find the time(s), if any, the object reaches a minimum displacement.



85. An object moves in a straight line with velocity $v(t) = 24t - 6t^2$.

a) How far does it travel in the first 4 seconds?

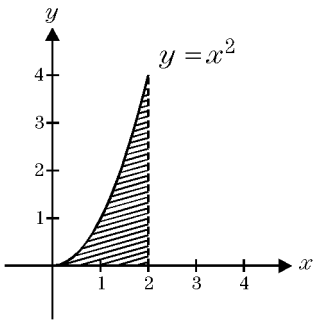
b) What is the *total* distance travelled by the object in the first 5 seconds?

86. On the planet Euclidean acceleration due to gravity is 16 m/sec. An object is fired vertically upward from ground level so that it takes 8 seconds before it returns to the ground.

a) What is the maximum height the object reaches?

b) What was the initial velocity with which the object was fired?

87. Which of the following definite integrals represents the area of the shaded region?



A) $\int_0^4 x^2\,dx$

B) $\int_0^2 x^2\,dx$

C) $\int_1^2 x^2\,dx$

D) $\int_0^2 x^2$

E) $\int_0^4 x^2$

88. Write the definite integral that represents the area of the region enclosed by $y = 9x - x^2$ and the x-axis.

A) $\int_0^9 (9x - x^2)\,dx$

B) $\int_0^3 (9x - x^2)\,dx$

C) $2\int_0^9 (9x - x^2)\,dx$

D) $\int_0^3 (9y - y^2)\,dy$

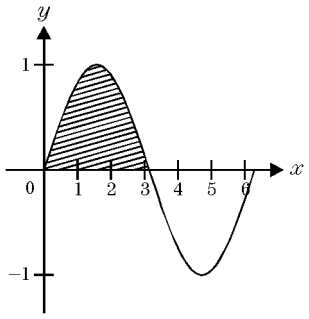
E) $2\int_0^3 (9y - y^2)\,dy$

89. Find the area of the region bounded by $y = 9 - 9x^2$ and $y = 0$.

90. Find the area of the region in the first quadrant bounded by the graph $y = e^{-2x}$ and the line $y = 0$.

91. Find the area of the region R bounded by the curve $y = \frac{1}{x}$, and the lines $x = 2$, $x = 6$, and $y = 0$.

92. Given the interval $[0, \frac{\pi}{3}]$, calculate the area between the curve $y = \sin x$ and the x-axis.



93. Which of the following would yield the area of the region bounded by the graphs of $y = 2x$ and $y = 11x - x^3$?

A) $\int_{-3}^3 (x^3 - 9x)\,dx$

B) $\int_{-3}^3 (9x - x^3)\,dx$

C) $2\int_0^3 (9x - x^3)\,dx$

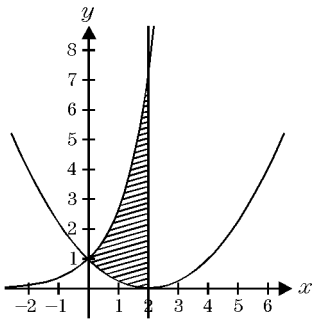
D) $\int_{-3}^0 (9x - x^3)\,dx + \int_0^3 (x^3 - 9x)\,dx$

E) $2\int_{-3}^3 (x^3 - 9x)\,dx$

94. Let R be a region in the first quadrant enclosed by the curves of $y = 10 - x^2$, $y = 3x$, and the y-axis.

Find the area of the enclosed region.

95. Find the area of R, the shaded region enclosed by the graphs of $f(x) = e^{2x}$, $g(x) = \frac{1}{2}(x-2)^2$, and the line $x = 2$.



96. Find the area of the region between the curves $f(x) = \cos x$, $f(x) = \sin x$, $x = 0$, and $x = \frac{\pi}{4}$.

97. Find the area (in square units) of the region bounded by $y = x^3 + 3x^2 + 3x$ and $y = x(x+6)$.

98. Find the area of region R in quadrant III bounded by $y = -x^4$ and $y = x^3$.

99. Find the area bounded between $y = e^x$, $y = \dfrac{3}{x}$, and $x = 3$.

100. Find the area of the region in the first quadrant between the curves $f(x) = \cos x, f(x) = \sin^3 x$, and $x = 0$.

101. A pyramid with a square base and congruent triangular sides is 5 m high. If each cross section of the pyramid is a square parallel to the base, then what is the volume of the pyramid?

102. A solid has a base bounded by $x^2 + y^2 = 25$. Find the volume of the solid if every plane section perpendicular to the diameter is a square.

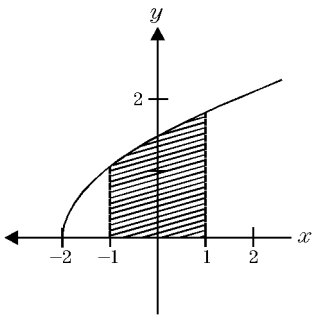
103. A half of a pepperoni stick is 10 cm long. Assume that a cross section perpendicular to the axis of the pepperoni at a distance x from the end is a circle of radius $\sqrt{3x}$. What is the volume of the pepperoni?

104. Find the volume of the solid of revolution obtained by rotating the region R bounded by $y = 3x$, $y = 0$, and $x = 2$ about the x-axis.

105. Let R be a region in the first quadrant enclosed by the curves of $y = (3 - x)(3 + x)$, $y = 8x$, and the y-axis. Find the volume of the solid generated by revolving the region R about the x-axis.

106. Find the volume of the solid of revolution formed by revolving the region bounded by $y = -x + 5$, $y = 0$, and $x = 0$ about the y-axis.

107. Find the volume of the solid formed by revolving $f(x) = (x + 2)^{1/2}$ about the x-axis on the interval $[-1, 0]$.



108. The region under $y = x^2 + 2$ is rotated about the x-axis. Find the volume of the solid over the interval $[-2, 2]$.

109. Find the volume of the solid formed by revolving the region bounded by $y = x^2$ and $y = 6$ about the x-axis.

110. Find the volume of the solid formed by revolving the region bounded by $y = e^x$, $y = 0$, $x = 0$ and $x = 1$ about the y-axis.

111. Which of the following yields the volume of the solid generated by revolving the region bounded by the graphs of $y = x^3$ and the line $y = x$, between $x = 0$ and $x = 3$, about the y-axis?

A) $\pi \int_0^3 (x^2 - x^4)\,dx$ B) $\pi \int_0^3 (y^{1/3} - y)^2\,dy$

C) $\pi \int_0^3 (x^4 - x^2)\,dx$ D) $\pi \int_0^3 (y^{2/3} - y^2)\,dy$

E) $2\pi \int_0^3 (x^{4/3})\,dx$

112. Find the volume of the solid formed by rotating the curve $y = x^3$, bounded by $y = 0$ and $x = 3$, about the y-axis.

113. Find the volume of the solid formed by rotating the area between $y = x^2$ and $y = 6x - x^2$ about the y-axis.

114. Find the volume of the solid formed by rotating the area between $y = x^2$ and $y = 6x - x^2$ about $x = 4$.

115. Write an equation for the amount P of a radioactive substance with a half-life of 11 days, if 7 grams are present when $t = 0$.

A) $Q(t) = 7e^{-0.0630t}$ B) $Q(t) = 7e^{5.5t}$

C) $Q(t) = 7e^{-7.6246t}$ D) $Q(t) = 7e^{38.5t}$

E) $Q(t) = 7e^{-0.6931t}$

116. In 1995 the population of a town was 35,000 and in 2001 it was 30,000. Assuming the population decreases continuously at a constant rate proportional to the existing population, estimate the population in the year 2010.

117. The population P of a suburb is given by $P = 5000e^{kt}$. Let $t = 0$ correspond to the year 1996 and suppose the population in 1990 was 3600. Use this model to predict the population in 2010.

118. The number N of bacteria in a culture is given by $N = 350e^{kt}$. If $N = 450$ when $t = 6$ hours, find k (to the nearest hundredth) and then determine approximately how long it will take for the number of bacteria to increase 5-fold in size.

119. Let R be the first quadrant region enclosed by the graphs of $y = e^{x-1}$ and $y = 5 - \cos x$.

a) Find the area of R.

b) Find the volume of the solid generated when R is revolved about the x-axis.

c) The region R is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is a square. Find the volume of this solid

120. Water is pumped into a tank at a constant rate of 6 gallons/min. Water leaks out of the tank at the rate of $\sqrt{t+3}$ gallons/min for $0 \le t \le 80$ minutes. At time $t = 0$, the tank contains 20 gallons of water.

a) How many gallons of water leak from time $= 0$ to $t = 6$?

b) How many gallons of water are in the tank at time $t = 6$ minutes?

c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t?

d) At what time t, for $0 \le t \le 80$ is the amount of water in the tank a maximum. Justify your answer.

121. Elvis drinks a 500 mL milkshake at Ye Olde Fountain Shoppe. Sadly, and unknown to Elvis, there is a crack in the glass that is getting wider with time, and the milkshake leaks out onto the counter at the rate $r(t) = \ln(t+1)$ mL/sec, for t seconds. Elvis drinks his milkshake at the constant rate of 10 mL/sec and begins drinking at the same time it starts leaking out (the instant he gets it), $t = 0$.

a) How fast is milkshake leaving the glass at time $t = 5$ seconds?

b) How much milkshake has left the mug at time $t = 5$ seconds?

c) Write an expression for $B(t)$, the volume of milkshake left in the glass at time t.

d) Elvis usually drinks a milkshake in 50 seconds. When will he actually finish it?

122. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{4x^3}{e^y}$

a) Find the solution $y = f(x)$ to the differential equation satisfying $f(0) = 1$.

b) Find the domain and range of the function found in part (a).

123. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{2t^2 - 3}{t}$ for $t > 0$.

a) In which direction, left or right, is the particle moving at $t = 0.5$? Why?

b) Find the acceleration of the particle at time $t = 0.5$. Is the velocity increasing at $t = 0.5$? Why or why not?

c) Given that $x(t)$ is the position of the particle at time t and that $x(1) = -1$, find $x(4)$.

d) Find the total distance traveled by the particle from $t = 1$ to $t = 4$.

124. The region R is bounded by $y = -(x-2)^2$, and $y = -9$.

a) Find the area of R

b) Find the volume of the solid generated by revolving R about the x-axis.

c) There is a number k, $k < -9$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Set up, but do not solve, the integral expression that can be used to find k.

125.

t (hours)	$R(t)$ (gallons/hr)
0	9.9
4	10.8
8	11.1
12	11.2
16	10.8
20	10.2
24	9.3

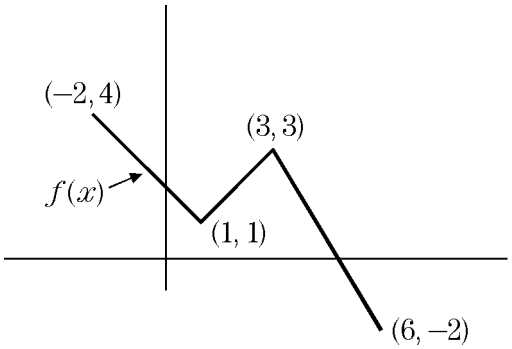
The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table shows the rate as measured every 4 hours for a 24-hour period.

a) Use a midpoint Riemann sum with 3 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

b) Is there some time t, $0 < t < 24$ such that $R'(t) = 0$? Justify your answer.

c) The rate of water flow, $R(t)$, can be approximated by $Q(t) = \frac{1}{85}(845 + 21t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour period. Indicate units of measure.

126.



The graph of the function, $f(x)$, consisting of three line segments is shown. Let $g(x) = \int_1^x f(t)\,dt$.

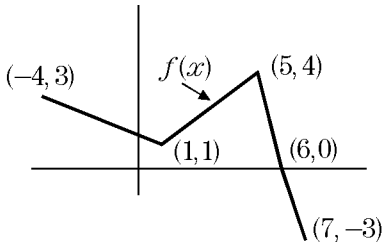
a) Compute $g(6)$ and $g(-2)$.

b) Find the instantaneous rate of change of g, with respect to x, at $x = 3$.

c) Find the absolute maximum value of g on the closed interval $[-2, 6]$. Justify your answer.

d) The second derivative of g is not defined at $x = 1$ and $x = 3$. How many of these values are x-coordinates of points of inflection of the graph of g. Justify your answer.

127.



The graph of the function, $f(x)$, consisting of four line segments is shown. Let $g(x) = \int_1^x f(t)\,dt$.

a) Compute $g(5)$ and $g(7)$.

b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

c) Find the absolute maximum value of g on the closed interval $[-4, 7]$. Justify your answer.

d) The second derivative of g is not defined at $x = 1, 5$ and $x = 6$. How many of these values are x-coordinates of points of inflection of the graph of g. Justify your answer.