SHEET #1200: PRACTICE FINAL TEST | TURN IN ALL WORK

PAGE 1. COMPREHENSIVE NAME $\boxed{\text{KEY}}$ v.2 v.3 Period:

31 PROBLEMS TOTAL. ANSWER TO 3 DECIMALS. TI-83/84 ALLOWED.
NOTE: SKIP PROBLEMS WITH X MARKED.

Directions: Show all steps leading to your answers, including any intermediate results obtained using a graphing utility. Use the back of the test or another sheet of paper if necessary.

1. Let $f(x) = \begin{cases} \dfrac{6x+4}{x-6}, & x < 1 \quad \text{USE FOR } x \to -\infty \\ 2 - x^3, & x \geq 1 \quad \text{USE FOR } x \to \infty \end{cases}$.

Find the limit of $f(x)$ as (a) $x \to -\infty$, (b) $x \to 1^-$, (c) $x \to 1^+$, and (d) $x \to \infty$

b) $\dfrac{6(1)+4}{1-6} = \dfrac{10}{-5} = -2$ c) $2 - 1^3 = 1$ d) $\lim_{x \to \infty} (-x^3)$

1. (a) $\lim_{x \to -\infty} \dfrac{6x}{x} = \boxed{6}$

(b) $\boxed{-2}$

(c) $\boxed{1}$

(d) $\boxed{-\infty}$

2. Find the equations of all lines tangent to $y = x^2 - 6$ that pass through the point (4, 6). LET $x = a$ AT POINT ON CURVE.

$f'(a) = 2a$
$f(a) = a^2 - 6$
SLOPE OF LINE = SLOPE OF CURVE.
$\dfrac{\Delta y}{\Delta x} = \dfrac{(a^2-6)-6}{a-4} = 2a$
$a^2 - 12 = 2a(a-4)$
$a^2 = 2a^2 - 8a + 12$
$0 = (a-2)(a-6)$
$a = 2, a = 6$
SLOPE $= 2a = \{4 \text{ or } 12\}$

2. $y - 6 = 4(x-4)$, $y - 6 = 12(x-4)$ $\boxed{y = 4x - 10 \quad y = 12x - 42}$

3. Find $\dfrac{dy}{dx}$, where $y = x^6 + 8x^2 - 11x$.

3. $\boxed{6x^5 + 16x - 11}$

4. Find $\dfrac{dy}{dx}$, where $y = \dfrac{\sin x}{x^2 + 3}$

$\dfrac{(\sin x)'(x^2+3) - (\sin x)(x^2+3)'}{(x^2+3)^2}$

4. $\boxed{\dfrac{(x^2+3)\cos x - 2x \sin x}{(x^2+3)^2}}$

5. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $4y(x^3 + 2) = 5y^3 + x$.

LEFT SIDE ⇒ PRODUCT RULE: $4y'(x^3+2) + 4y(3x^2) = 15y^2y' + 1$
$12x^2y - 1 = y'(15y^2 - 4x^3 - 8)$

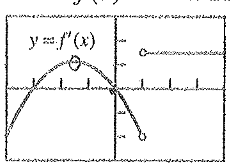
5. $y' = \boxed{\dfrac{12x^2y - 1}{15y^2 - 4x^3 - 8}}$

6. Find $\dfrac{dy}{dx}$ if $y = 3^{2.5x}$.

(A) $(\ln 2.5)3^{2.5x}$ (B) $2.5(3^{2.5x})$ (C) $(\ln 3)(3^{2.5x})$
(D) $2.5(\ln 3)(3^{2.5x})$ (E) $(2.5x)(3^{1.5x})$
↑ CHAIN RULE

6. $\boxed{D}$ $2.5 \ln(3)\, 3^{2.5x}$

7. The graph below represents the *derivative* of a function f, where $f(1) = -1$. Sketch a possible graph of $y = f(x)$.

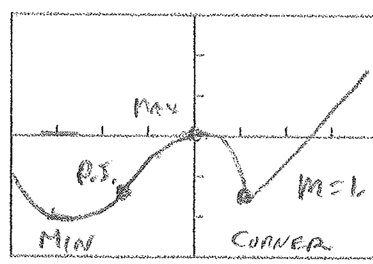


[-4, 4] by [-3, 3]

ARBITRARILY PICK (0,0) TO BE ON GRAPH.

x	y'		
-3	0	MIN	f INCREASING
-1.5	1	P.I.	
0	0	MAX	f DECREASING
1	DNE	CORNER	
2	1.5		f INCREASING

7.



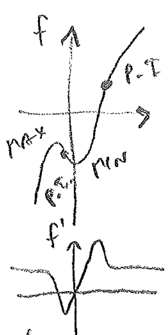
[-4, 4] by [-3, 3]

8. For $y = 2x - 5e^{-x^2}$, use graphing techniques with analytical support to find the approximate intervals on which the function is $f' = 2 + 10xe^{-x^2}$

(a) increasing, (b) decreasing,
(c) concave up, (d) concave down.

Then find any

(e) local extreme values, (f) inflection points.



ZEROS OF f'
$(-1.393, -3.504)$
$(-0.209, -5.204)$

ZEROS OF f'' = MAX/MIN ON f'
$(0.707, -1.618)$
$(-0.707, -4.447)$
CONCAVE UP: f' INCREASING.

8. (a) $(-\infty, -1.393)$ AND $(-0.209, \infty)$
(b) $(-1.393, -0.209)$
(c) $(-0.707, 0.707)$
(d) $(-\infty, -0.707)$ AND $(0.707, \infty)$
(e) Min: -5.204 at $x \approx -0.209$
Max: -3.504 at $x \approx -1.393$
(f) $(-0.707, -4.447)$, $(0.707, -1.618)$

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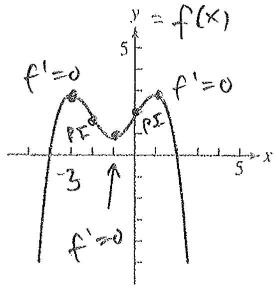
V. 2
V. I

9. Use the graph of the function f to estimate where (a) f' and (b) f'' are 0, positive, and negative.



CONCAVE UP.

9. (a) $f' = 0$: $x = -3, -1, 1$
$f' > 0$: $x < -3, \quad -1 < x < 1$
$f' < 0$: $-3 < x < -1, \quad x > 1$
(b) $f'' = 0$: $x = -2, 0$
$f'' > 0$: $-2 < x < 0$
$f'' < 0$: $x < -2, \quad x > 0$

10. You are planning to make an open box from a 24- by 32-inch piece of sheet metal by cutting congruent squares from the corners and folding up the sides. You want the box to have the largest possible volume.

(a) What size square should you cut from each corner? (Give the side length of the square.)
(b) What is the largest possible volume your box will have?

10. (a) $\boxed{4.526 \text{ in}}$
(b) $\boxed{1552.539 (\text{in})^3}$

32 in, 24 in, x

$V = H \cdot L \cdot W$
$V = x(32-2x)(24-2x)$
MAXIMIZE V.

11. Find the linearization $L(x)$ of $f(x) = 2x^4 - 40x^2 + 100$ at $x = 3$.

(A) $L(x) = -24x$
(B) $L(x) = -24x - 26$
(C) $L(x) = -24x - 98$
(D) $L(x) = -98x$
(E) $L(x) = -21x - 35$

11. $f'(x) = 8x^3 - 80x$
$f(x) \approx f(3) + f'(3)(x-3)$
$L(x) = -98 + (-24)(x-3)$
$\boxed{L(x) = -24x - 26}$ $\boxed{B}$

X **12.** You are using Newton's method to solve $e^x - 2$. If your first guess is $x_1 = 1$, what value will you calculate for the next approximation x_2?

X **12.**

13. Suppose that the edge lengths x, y, and z of a closed rectangular box are changing at the following rates:
$\frac{dx}{dt} = 2$ ft/sec, $\frac{dy}{dt} = -5$ ft/sec, $\frac{dz}{dt} = 0$ ft/sec.
Find the rates at which the box's (a) volume, (b) surface area, and (c) diagonal length $s = \sqrt{x^2 + y^2 + z^2}$ are changing at the instant when $x = 7$ ft, $y = 4$ ft, and $z = 9$ ft.

13. (a) $2 \cdot 4 \cdot 9 - 7 \cdot 5 \cdot 9 = \boxed{-243 \text{ ft}^3/\text{sec}}$
(b) $2(2 \cdot 4 - 7 \cdot 5 + 2 \cdot 9 - 5 \cdot 9) = \boxed{-108 \text{ ft}^2/\text{sec}}$
(c) $\frac{1}{\sqrt{146}}(7 \cdot 2 - 4 \cdot 5) = -6/\sqrt{146} \approx \boxed{-0.497 \text{ ft}/\text{sec}}$

13.
a, $(xyz)' = x'yz + xy'z + \cancel{xyz'} = 2 \cdot 4 \cdot 9 + 7(-5)9 + \cancel{7 \cdot 4 \cdot 0}$
b, $(2xy + 2xz + 2yz)' = 2[x'y + xy' + x'z + \cancel{xz'} + y'z + \cancel{yz'}]$
$= 2(2 \cdot 4 + 7(-5) + 2(9) + (-5)9)$
c, $(\sqrt{x^2+y^2+z^2})' = \frac{1}{2s}(2xx' + 2yy' + \cancel{2zz'}) = \frac{7 \cdot 2 + 4 \cdot (-5) + \cancel{9 \cdot 0}}{\sqrt{7^2+4^2+9^2}}$

14. The table below shows the velocity of a running dog during a 35-second time interval. Use the right-endpoint values (RRAM) to estimate the distance traveled, using 7 intervals of length 5.

14. $\boxed{930 \text{ ft}}$

Time (sec)	0	5	10	15	20	25	30	35
Velocity (ft/sec)	18	22	28	27	25	26	28	30

$\text{dist} = \Delta s = \sum v_i \Delta t = (22+28+27+25+26+28+30) \cdot 5 = 930$ ft.
RIGHT END POINTS DO NOT USE (0, 18).

2.

15. Express the limit as a definite integral.

$\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(5c_k^{\,2} - \frac{3}{c_k}\right) \Delta x$, where P is any partition of $[3, 11]$.

15. $\boxed{\int_3^{11} \left(5x^2 - \frac{3}{x}\right) dx}$

16. Consider the integral $\int_0^5 (x^2 + 2x - 4)\,dx$.

(a) Estimate the value of the integral using 5 left-endpoint rectangles (LRAM).
(b) Estimate the value of the integral using the Trapezoidal Rule with $n = 5$.
(c) Integrate to find the exact value of the integral.

x	y	$\frac{1}{2}(y_i + y_{i+1})$
0	-4	-2.5
1	-1	1.5
2	4	7.5
3	11	15.5
4	20	25.5
5	31	
Σ	30	47.5

$\Delta x = 1$

16. (a) $\boxed{30}$
(b) $95/2 = \boxed{47.5}$
(c) $140/3 = \boxed{46.667}$

$\int_a^b f(x)\,dx = F(b) - F(a) = \left[\frac{x^3}{3} + x^2 - 4x\right]_0^5 = \frac{125}{3} + 25 - 20 = 140/3$

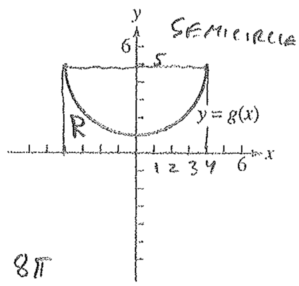
17. Suppose that f and g are continuous and that $\int_{-2}^{5} f(x)\,dx = 3$, $\int_{-2}^{3} f(x)\,dx = 7$, and $\int_{-2}^{3} g(x)\,dx = -8$. Find each integral.

(a) $\int_3^5 f(x)\,dx =$ (b) $\int_{-2}^{3} [f(x) + g(x)]\,dx$

(c) $\int_{-2}^{3} [4g(x) - 5f(x)]\,dx$

17. (a) $\int_3^5 f\,dx = \int_3^{-2} f\,dx + \int_{-2}^{5} f\,dx = -7 + 3 = \boxed{-4}$
(b) $7 - 8 = \boxed{-1}$
(c) $4(-8) - 5\cdot(7) = \boxed{-67}$

18. Find the average value of the function $g(x) = 5 - \sqrt{16 - x^2}$ on the interval $[-4, 4]$ without integrating, by appealing to the region between the graph and the x-axis.



$A_R = \square - \bigtriangledown = 8 \cdot 5 - \frac{1}{2}\pi(4)^2 = 40 - 8\pi$

18. $\boxed{5 - \pi}$

$\text{av}(g) = \frac{\int_{-4}^{4} g(x)\,dx}{4 - (-4)} = \frac{A_R}{8} = \frac{40 - 8\pi}{8} = 5 - \pi$

19. Use NINT to evaluate $\int_{-5.2}^{1.2} \frac{2e^x}{x^2 + \cos x}\,dx$.

fn/NT = TI-83.

19. $\boxed{4.985}$

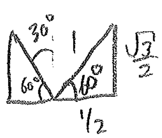
20. Find $\frac{d}{dx}\int_0^{3x} (2t - 7)\,dt$. $= \underbrace{[2(3x) - 7]}_{f(3x)} \underbrace{(3)}_{\text{CHAIN}=(3x)'}$

20. $\boxed{18x - 21}$

21. Evaluate each integral using Part 2 of the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{4} (5x^2 + 14x - 3)\,dx$ (b) $\int_1^8 x^{2/3}\,dx$

(c) $\int_{\pi/3}^{2\pi/3} \csc^2\theta\,d\theta = -\cot\theta\Big]_{\pi/3}^{2\pi/3} = -\frac{\cos\theta}{\sin\theta}\Big]_{\pi/3}^{2\pi/3} = -\left[\frac{-1/2}{\sqrt{3}/2} - \frac{1/2}{\sqrt{3}/2}\right] = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$



21. (a) $\left[5\frac{x^3}{3} + 7x^2 - 3x\right]_{-2}^{4} = \frac{620}{3} - \frac{62}{3} = \boxed{186}$
(b) $\left[\frac{3}{5}x^{5/3}\right]_1^8 = \frac{3}{5}(2^5 - 1) = \boxed{\frac{93}{5}} = 18.6$
(c) $\boxed{2/\sqrt{3}} \approx 1.155$

22. Find $\frac{dy}{dx}$ if $y = \int_1^{x^3} (6t^2 - 7)\,dt$.

(A) $18x^4 - 21x^2$ (B) $12x^5 - 7x^3$ (C) $12x^5 - 21x^2$
(D) $6x^6 - 7$ (E) $18x^8 - 21x^2$

22. $\boxed{E}$

$[6(x^3)^2 - 7](3x^2) = 18x^8 - 21x^2$, E.

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4. (continued) NAME V.3

$T_n = \frac{1}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)\cdot \Delta x$ Δx

23. Use the Trapezoidal Rule with $n = 6$ to approximate the value of $\int_2^4 5 \ln x\, dx$.

23. $\boxed{10.783}$

$\Delta x = \frac{4-2}{6} = \frac{2}{6} = \frac{1}{3}$ $T_6 = \frac{1}{2}(5)\left[\ln(2) + 2\ln(\frac{7}{3}) + 2\ln(\frac{8}{3}) + 2\ln(3) + 2\ln(\frac{10}{3}) + 2\ln(\frac{11}{3}) + \ln(4)\right]\cdot\frac{1}{3}$

$\approx \frac{1}{2}(5)(12.93943)(\frac{1}{3})$

≈ 10.783

24. Use the Fundamental Theorem of Calculus to evaluate $\int \sin\sqrt{x+7}\, dx.$ $\underset{\text{BY DEFINITION}}{=} F(x) + C_2$ where C_2 = CONSTANT and $F'(x) = \sin\sqrt{x+7}$.

24. $\boxed{E}$

(A) $-\cos x\sqrt{x+7} + C$

(B) $\int_0^x \frac{\cos\sqrt{t+7}}{2\sqrt{t+7}}\, dt + C$

(C) $\frac{\cos\sqrt{x+7}}{2\sqrt{x+7}} + C$

(D) $\int_x^0 \sin\sqrt{t+7}\, dt + C$

(E) $\int_0^x \sin\sqrt{t+7}\, dt + C$ $= F(x) - F(0) + C$
$= F(x) + C_2$ where $C_2 = -F(0) + C$ = CONSTANT

25. Evaluate the integral.

$\int\left(3x^5 - \frac{1}{x^3} + e^{7x}\right) dx = \frac{3x^6}{6} - \frac{x^{-2}}{-2} + \frac{e^{7x}}{7} + C$

25. $\boxed{\frac{x^6}{2} + \frac{1}{2x^2} + \frac{1}{7}e^{7x} + C}$

$y = \int(-5x+7)^2 dx = \frac{(-5x+7)^3}{3(-5)} + C$ "CHAIN RULE"

26. Solve the initial value problem.

$\frac{dy}{dx} = (-5x + 7)^2,\ y(1) = -1$

METHOD 1: SOLVE FOR C.

$-1 = -\frac{(-5\cdot 1+7)^3}{15} + C$ $-1 = \frac{-2^3}{15} + C$

$C = -15/15 + 8/15 = \frac{-7}{15}$

26. $\boxed{-\frac{(-5x+7)^3}{15} - 7/15}$

METHOD 2: FTC

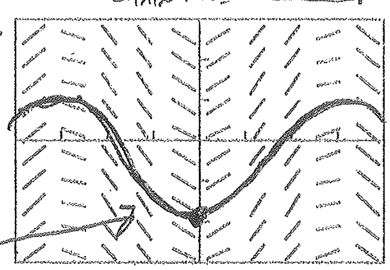
$y(x) = y(1) + \int_1^x (-5t+7)^2 dt = -1 + \left[\frac{(-5t+7)^3}{3(-5)}\right]_1^x = -1 - \frac{(-5x+7)^3}{15} + \frac{(-5\cdot 1+7)^3}{15}$ "CHAIN RULE"

$= -1 - \frac{(-5x+7)^3}{15} + 8/15$

$= -\frac{(-5x+7)^3}{15} - \frac{7}{15}.$

27. Suppose $y = f(x)$ is a solution to the differential equation whose slope field is shown. Sketch a possible graph for the function f satisfying $f(0) = -2$.

27.



(0, −2)

[−4, 4] by [−3, 3]

28. Use substitution to evaluate the integral.

$\int \sec^5 x \tan x\, dx = \int \sec^4 x\,(\sec x\cdot\tan x)dx$

$d(\sec x)/dx = \sec x\cdot\tan x.$

28. $\boxed{\frac{\sec^5 x}{5} + C}$

29. Solve the differential equation by separation of variables.

$\frac{dy}{dx} = \frac{x+7}{3y^2\sin(y^3)}$

29. $\boxed{y = \sqrt[3]{\cos^{-1}(-x^2/2 - 7x - C)}}$

$\int \sin(y^3)\cdot 3y^2 dy = \int(x+7)dx$

$\boxed{-\cos(y^3)} = \frac{x^2}{2} + 7x + C$

$y^3 = \cos^{-1}(-(x^2/2 + 7x + C))$

$y = [\cos^{-1}(-(x^2/2+7x+C))]^{1/3}$

X **30.** Use integration by parts to evaluate $\int x\cos(3x + 4)\, dx.$ X 30.

31. Use tabular integration or another method to evaluate the integral.
$\int x^2 e^{-5x}\,dx$

31. ______

32. Suppose a certain element has a half-life of 6.3 days. If a sample contains 700 grams of this element, how much of it will remain after 16 days?

32. $\boxed{120.386\text{ g}}$

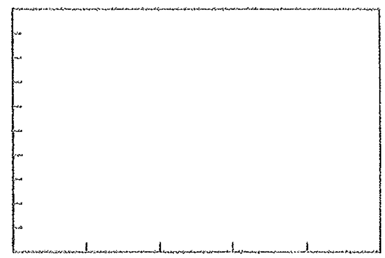
$y = A\left(\frac{1}{2}\right)^{t/T} = 700\left(\frac{1}{2}\right)^{6.3/16}$

33. The table gives the growth of a population of coyotes. Let $x = 0$ represent 1950, $x = 10$ represent 1960, and so forth.

Year	1950	1960	1970	1980	1990
Population	23	102	325	553	642

(a) Find the logistic regression equation for the data and superimpose its graph on a scatter plot of the data.
(b) Find the carrying capacity predicted by the regression equation.
(c) Find when the rate of growth predicted by the regression equation changes from increasing to decreasing. Estimate the population at this time.

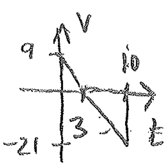
33. (a)



[0, 50] by [0, 1000]

(b) ______
(c) Year: ______
Population: ______

34. The function $v(t) = 9 - 3t$ is the velocity in ft/sec of a particle moving along the x-axis for $0 \le t \le 10$. Use analytic methods to answer the following questions.



TWO TRIANGLES

(a) Determine when the particle is moving to the right, to the left, and stopped. $V = 0$ FOR $t = 3$
(b) Find the particle's displacement for the given time interval. $\frac{1}{2}(9\cdot 3) + \frac{1}{2}(-21\cdot 7) = 13.5 - 73.5 = -60$ ft
(c) Find the total distance traveled by the particle.
$13.5 + 73.5 = 87$ ft.

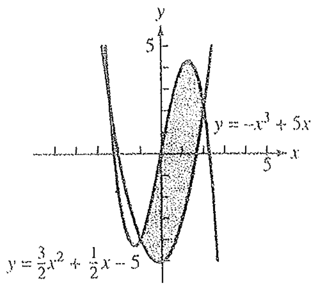
34. (a) Right: $0 \le t < 3$ sec
Left: $3 < t \le 10$ sec
Stopped: $t = 3$ sec
(b) -60 ft
(c) 87 ft

35. At a certain glue factory, the production rate in gallons per hour x hours after the factory opens in the morning is given by the function
$r(x) = -\frac{1}{2}x^4 + 12x^3 - 105x^2 + 400x$. How much glue is produced during the first 5 hours after opening?

$R(5) = \int_0^5 r(x)\,dx = 2187.5$ gal.

35. $\boxed{2187.5\text{ gal.}}$

36. Find the area of the shaded region analytically. **36.** $\frac{27}{2} = \boxed{13.5 \text{ UNITS}^2}$



INTERSECTIONS

$-x^3+5x = \frac{3}{2}x^2 + \frac{1}{2}x - 5$

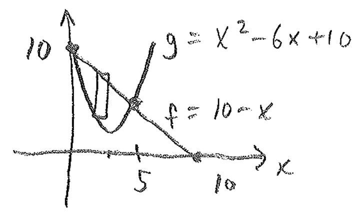
$(-1,-4)$ AND $(2,2)$

$A = \int_{-1}^{2} \left[(-x^3+5x) - (\frac{3}{2}x^2 + \frac{1}{2}x - 5)\right] dx = 13.5 \text{ UNITS}^2$

X **37.** A curve is given by $y = f(x)$ for $a \le x \le b$, where $a > 0$ and $f(x) > 0$. The integral $\int_a^b 2\pi x \cdot f(x)\,dx$ can be used to find which of the following? X **37.** ____

(A) The length of the curve
(B) The volume of the solid generated by revolving the the region below the curve about the x-axis
(C) The volume of the solid generated by revolving the region below the curve about the y-axis
(D) The area of the surface generated by revolving the curve about the x-axis
(E) The area of the surface generated by revolving the curve about the y-axis

QUESTION 38.



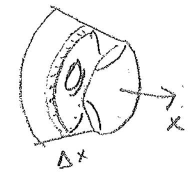
INTERSECTION $f = g$

$10 - x = x^2 - 6x + 10$

$0 = x^2 - 5x = (x-5)x$.

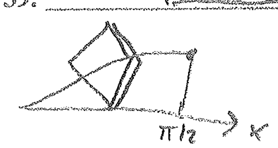
$x = 0,\ x = 5$

$V_{WASHER} = \pi (f - g)^2 \Delta x$



38. A region is bounded by the line $y = 10 - x$ and the parabola $y = x^2 - 6x + 10$. Find the volume of the solid generated by revolving the region about the x-axis. **38.** $\pi \int_0^5 (10-x)^2 - (x^2 - 6x + 10)^2 dx = \boxed{\frac{625\pi}{3} \text{ UNITS}^3}$ $\approx 208.333\pi$ ≈ 654.498

39. The base of a solid is the region between the x-axis and the graph of $y = \sqrt{\sin^3 x}$ for $0 \le x \le \frac{\pi}{2}$. Each cross section perpendicular to the x-axis is a rectangle whose base is in the xy-plane and whose other side has length $\cos x$. Find the volume of the solid. **39.** $\boxed{\frac{2}{5} \text{ UNITS}^3}$



$\text{VOLUME}_{\text{RECTANGLE}} = \sqrt{\sin^3 x} \cdot \cos x \, \Delta x$

$V = \int_0^{\pi/2} \sin^{3/2} x \cos x \, dx = \frac{2(\sin x)^{5/2}}{5} \Big]_0^{\pi/2} = \frac{2}{5} 1^{5/2} - 0 = \frac{2}{5} \text{ UNITS}^3$.

"CHAIN RULE"

X **40.** Find the length of the curve described by $y = 2x^{3/2}$ for $0 \le x \le 7$. X **40.** ____

X **41.** A spring has a natural length of 14 cm. A 24-N force stretches the spring to 17 cm. How much work is done in stretching the spring from 14 cm to 20 cm? X **41.** ____

DON'T PANIC!