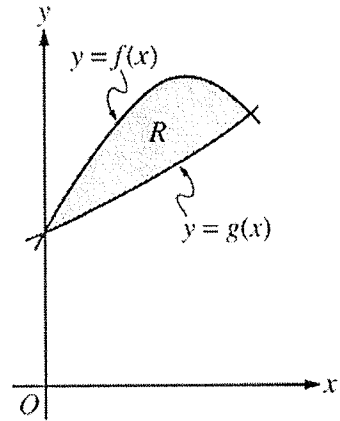


AB 2005B Question 1

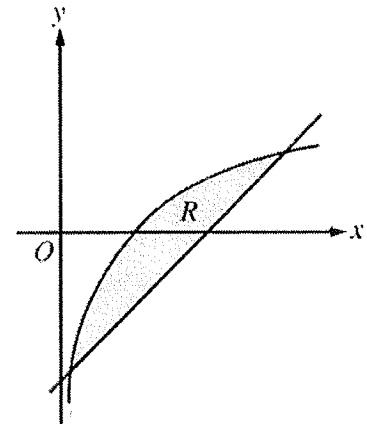
Let  $f$  and  $g$  be the functions given by  $f(x) = 1 + \sin(2x)$  and  $g(x) = e^{x/2}$ . Let  $R$  be the shaded region in the first quadrant enclosed by the graphs of  $f$  and  $g$  as shown in the figure above.



- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.
- (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles with diameters extending from  $y = f(x)$  to  $y = g(x)$ . Find the volume of this solid.

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Let  $R$  be the shaded region bounded by the graph of  $y = \ln x$  and the line  $y = x - 2$ , as shown above.



- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -3$ .
- (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when  $R$  is rotated about the  $y$ -axis.

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The graphs of  $f$  and  $g$  intersect in the first quadrant at  $(S, T) = (1.13569, 1.76446)$ .

↑  $T$  not needed

$$\begin{aligned} \text{(a) Area} &= \int_0^S (f(x) - g(x)) dx \\ &= \int_0^S (1 + \sin(2x) - e^{x/2}) dx \\ &= 0.429 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^S ((f(x))^2 - (g(x))^2) dx \\ &= \pi \int_0^S ((1 + \sin(2x))^2 - (e^{x/2})^2) dx \\ &= 4.266 \text{ or } 4.267 \end{aligned}$$

$$\begin{aligned} \text{(c) Volume} &= \int_0^S \frac{\pi}{2} \left( \frac{f(x) - g(x)}{2} \right)^2 dx \\ &= \int_0^S \frac{\pi}{2} \left( \frac{1 + \sin(2x) - e^{x/2}}{2} \right)^2 dx \\ &= 0.077 \text{ or } 0.078 \end{aligned}$$

1 : correct limits in an integral in (a), (b), or (c)

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ (-1) \text{ each error} \\ \text{Note: } 0/2 \text{ if integral not of form} \\ \int_a^b (R^2(x) - r^2(x)) dx \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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$\ln(x) = x - 2$  when  $x = 0.15859$  and  $3.14619$ .  
Let  $S = 0.15859$  and  $T = 3.14619$

$$\text{(a) Area of } R = \int_S^T (\ln(x) - (x - 2)) dx = 1.949$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_S^T ((\ln(x) + 3)^2 - (x - 2 + 3)^2) dx \\ &= 34.198 \text{ or } 34.199 \end{aligned}$$

$$\text{(c) Volume} = \pi \int_{S-2}^{T-2} ((y+2)^2 - (e^y)^2) dy$$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

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3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$