Name KEY

Sheet #1150 - Ch7, 8, 11 Review

Per/Sec. ____________

Note: Make sure you can do each question with or without multiple choices given. Show setup of integrals or sums. Be prepared to do each problem without the calculator (when the integral is difficult, showing a set-up would suffice).

1. Solve the differential equation $\dfrac{dy}{y^2} = x\,dx$.

A) $x^2y = C$ B) $xy^2 = C$ C) $x^2 + y = C$

D) $3y + x^2 = C$ E) $x^2 + \dfrac{2}{y} = C$

$-\dfrac{1}{y} = \dfrac{x^2}{2} + C_1$ FROM INTEGRATING EACH SIDE.

$-\dfrac{2}{y} = x^2 + C_2$ $\boxed{x^2 + 2/y = C}$ NOTE: $1/y^2 = y^{-2}$

2. Solve the differential equation $e^{x+y}\,dy = dx$.

A) $e^{-x} + e^{y} = C$ B) $e^{-x} + e^{-y} = C$

C) $e^{x} + e^{y} = C$ D) $e^{x} + e^{-y} = C$

E) $e^{-x} + 2e^{-y} = C$

$e^x \cdot e^y\,dy = dx$

$\int e^y\,dy = \int \dfrac{dx}{e^x}$

$\int e^y\,dy = \int e^{-x}\,dx$ → $\boxed{e^y = -e^{-x} + c}$

CHECK: $e^y \dfrac{dy}{dx} = e^{-x}$ (BY IMPLICIT DIFFERENTIATION)

3. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{2x^3}{e^{3y}}$

Find the solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

$\int e^{3y}\,dy = \int 2x^3\,dx$

$+\dfrac{1}{3}e^{3y} = \dfrac{2x^4}{4} + C_1$

$e^{3y} = \dfrac{3}{2}x^4 + c$

PUT IN POINT ALREADY HERE $(0, 1/2)$.

$e^{3/2} = 0 + c$

$\boxed{y = \frac{1}{3}\ln\left(\frac{3}{2}x^4 + e^{3/2}\right)}$

4. a) Which of the following differential equations goes with the slope field shown?

A) $y' = x - y$

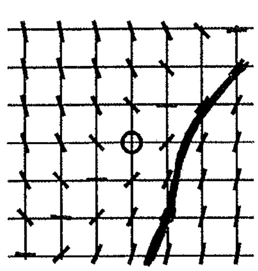
B) $y' = x + y$

C) $y' = x^2 + y^2$

D) $y' = \dfrac{x}{y}$

E) $y' = -\dfrac{x}{y}$

y' HAS TO BE 0 AT $(1,1)$, 1 AT $(1,0)$ SO ITS A



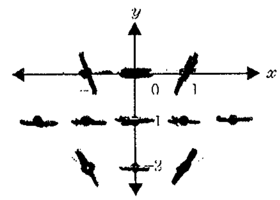
$[-3,3] \times [-3,3]$

b) Make a sketch of the solution to the differential equation whose graph passes through the point $(1,-2)$.

5. a) Given the differential equation $\dfrac{dy}{dx} = 2x(y+1)^2$, sketch its slope field at the eleven points indicated.

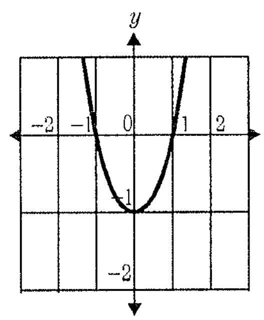
IF $x = 0$, $\dfrac{dy}{dx} = 0$

IF $y = -1$, $\dfrac{dy}{dx} = 0$



x	y	dy/dx
1	0	2
-1	0	-2
1	-2	2
-1	-2	-2

b) Use the slope field from part (a) to explain why a solution such as the one shown below is impossible.



SOLUTION IS NOT $y = x^2 - 1$.

$\int \dfrac{dy}{(y+1)^2} = \int 2x\,dx$

$\dfrac{-1}{(y+1)} = x^2 + c$

$\dfrac{-1}{x^2+c} - 1 = y$

6. Solve $yy' = 8xe^{2x^2}$, INITIAL COND: $y(0) = 1$.

$y\dfrac{dy}{dx} = 8xe^{2x^2}$

$y\,dy = 8xe^{2x^2}\,dx$

$\int y\,dy = \int 8xe^{2x^2}\,dx$ $u = 2x^2$, $du = 4x\,dx$

$\dfrac{y^2}{2} = 2\int e^u\,du$

$\dfrac{y^2}{2} = 2e^u + C_1$

$\dfrac{y^2}{2} = 2e^{2x^2} + C_1$

$\dfrac{y^2}{2} = 2e$

$y = \sqrt{4e^{2x^2} + C}$

$1 = y(0)$.

$1 = \sqrt{4e^0 + C}$

$C = -3$

$\boxed{y = \sqrt{4e^{2x^2} - 3}}$

7. $\int \cos\frac{x}{3}\,dx =$

A) $\sin\frac{x}{3}+C$ B) $\frac{1}{3}\sin(3x)+C$

C) $-3\sin\frac{x}{3}+C$ D) $3\sin\frac{x}{3}+C$

E) $-\frac{1}{3}\sin\frac{x}{3}+C$

$u = \frac{x}{3}$ $\int \cos(u)\cdot 3du =$

$\frac{du}{dx} = \frac{1}{3}$ $dx = 3du$ $= 3\sin(u) + c$

$\boxed{= 3\sin(x/3) + c}$

8. $\int \frac{\ln(5x)}{x}\,dx =$

A) $\frac{1}{5}\ln 5x - x + C$ B) $\frac{1}{2}(\ln 5x)^2 + C$

C) $2x\ln 5x - x + C$ D) $\frac{1}{5}\ln\frac{1}{5}x + C$

E) $5x\ln 5x + C$

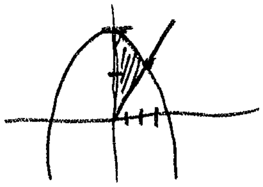
$u = \ln(5x)$ | $\int u\,du = \frac{u^2}{2} + c$

$du = \frac{1}{x}dx$ | $\boxed{= \frac{[\ln(5x)]^2}{2} + c}$

9. Let R be a region in the first quadrant enclosed by the curves of $y = 10 - x^2$, $y = 3x$, and the y-axis.

Find the area of the enclosed region.

A) $\frac{17}{3}$ B) $\frac{9}{2}$ C) $\frac{34}{3}$ D) 5 E) 9



$10 - x^2 = 3x$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$\boxed{x = 2 \quad y = 6}$ INTERSECT PT.

$A = \int_0^2 [(10-x^2) - 3x]\,dx = 10x - \frac{x^3}{3} - \frac{3x^2}{2}\Big]_0^2$

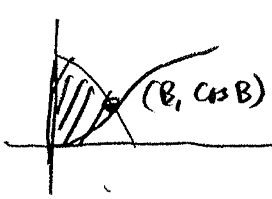
$= 20 - \frac{8}{3} - \frac{12}{2} = \frac{120}{6} - \frac{16}{6} - \frac{36}{6} = \frac{120-52}{6} = \frac{68}{6} = \boxed{\frac{34}{3}}$

10. Find the area of the region in the first quadrant between the curves $f(x) = \cos x$, $f(x) = \sin^3 x$, and $x = 0$.

A) 0.581 B) 0.663 C) 0.746

D) 0.792 E) 0.847

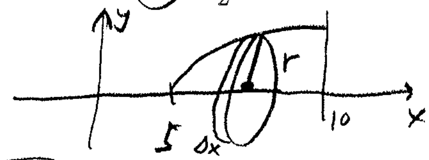
$B \approx 0.97203$



$A = \int_0^B (\cos x - \sin^3 x)\,dx = \boxed{0.663}$

11. Find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x-5}$, $y = 0$, and $x = 10$ about the x-axis. SHIFT RIGHT

A) 10π B) $\frac{25\pi}{2}$ C) 25π D) $\frac{75\pi}{2}$ E) 15π



$\boxed{\text{DISKS}}$ $V = \pi r^2 \Delta x = \pi(\sqrt{x-5})^2\Delta x$

$V = \int_5^{10} \pi(x-5)\,dx = \pi\left[\frac{x^2}{2} - 5x\right]_5^{10} =$

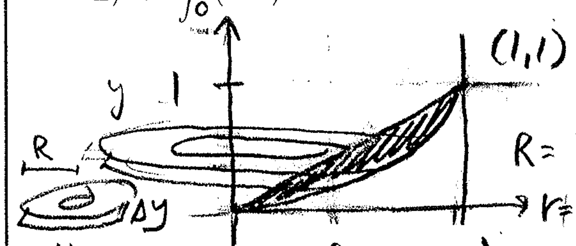
$\pi\left(\frac{100}{2} - 50 - \frac{25}{2} + 25\right) = \boxed{25\pi/2}$

12. Which of the following yields the volume of the solid generated by revolving the region bounded by the graphs of $y = x^3$ and the line $y = x$, between $x = 0$ and $x = 1$ about the y-axis?

A) $\pi\int_0^1 (x^2 - x^4)\,dx$ B) $\pi\int_0^1 (y^{1/3} - y)^2\,dy$

C) $\pi\int_0^1 (x^4 - x^2)\,dx$ D) $\pi\int_0^1 (y^{2/3} - y^2)\,dy$

E) $2\pi\int_0^1 (x^{4/3})\,dx$



$R = x = \sqrt[3]{y}$

$r = x = y$

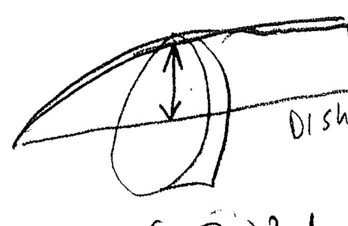
$\boxed{\text{WASHERS}}$ $V = (\pi R^2 - \pi r^2)\Delta y = \pi((y^{1/3})^2 - y^2)\Delta y$

$V = \int_0^1 \pi(y^{2/3} - y^2)\,dy$

13. A half of a pepperoni stick is 10 cm long. Assume that a cross section perpendicular to the axis of the pepperoni at a distance x from the end is a circle of radius $\sqrt{3x}$. What is the volume of the pepperoni?

A) 175π units^3 B) 300 units^3 C) 150π units^3

D) $\frac{167}{3}$ units^3 E) 75π units^3



$V = \pi\cdot(y)^2\cdot\Delta x$

DISKS.

$V = \int_0^{10} \pi(\sqrt{3x})^2\,dx = \int_0^{10} \pi\cdot 3x\,dx$

$= \pi\frac{3x^2}{2}\Big|_0^{10} = \frac{\pi\cdot 300}{2} = \boxed{150\pi}$

14. A mold culture doubles its mass every seven days. Find the growth model for a plate seeded with 0.9 grams of mold. $y_0 = 0.9.\ y = y_0 e^{kt}$

A) $y = 0.9e^{0.09902t}$ B) $y = 0.9e^{0.12183t}$
C) $y = 0.9e^{0.38541t}$ D) $y = 0.9e^{0.45128t}$
E) $y = 0.9e^{0.81818t}$

METHOD 1.

$(2)^{t/7} = e^{kt}$

$2^{1/7} = e^k$

$\ln(2^{1/7}) = k$

$= \frac{1}{7}\ln(2) = \boxed{0.09902}$

METHOD 2

$2 = e^{k\cdot 7}$

$\ln(2) = k\cdot 7$

$k = \ln(2)/7.$

$= 0.09902.$

15. In 1995 the population of a town was 35,000 and in 2001 it was 30,000. Assuming the population decreases continuously at a constant rate proportional to the existing population, estimate the population in the year 2010.

A) 20,143 B) 22,982 C) 21,327
D) 23,807 E) 51,439

$\frac{dP}{dt} = -kP \rightarrow P = P_0 e^{-kt}$

$30000 = 35000 \cdot e^{-k\cdot 6}$

$k = -\ln(6/7)/6 = 0.025692$

$P(15) = 35000 \cdot e^{-k\cdot 15} = \boxed{23{,}806.8}$

16. Use a Trapezoidal approximation for $\int_2^4 x^4\,dx$ with $n = 4$.

A) 210.8 B) 195.5 C) 200.6
D) 208.4 E) 203.1

X 2 2.5 3 3.5 4

$y_0\ y_1\ y_2\ y_3\ y_4$

$\Delta X = \frac{4-2}{4} = 0.5$

$T = (\Sigma\ \text{TRAPEZOIDS}) =$

$= \frac{1}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4)0.5$

$= \boxed{203.0625}$

17. The following table shows selected coordinates for $y = f(x)$:

x	100	200	300	400
y	1.2	2.3	2.5	4.9

Given that f is continuous on $[100, 400]$, find a trapezoidal approximation, with $n = 3$, for the area under the curve from $x = 1$ to $x = 4$. $\Delta X = \frac{400-100}{3} = 100$

$T = \frac{1}{2}(y(100) + 2y(200) + 2y(300) + y(400))100$

$= \frac{1}{2}(1.2 + 2\cdot 2.3 + 2\cdot 2.5 + 4.9)\cdot 100$

$= \frac{1}{2}(1.2 + 4.6 + 5 + 4.9)\cdot 100 = \frac{1}{2}15.7\cdot 100 = \frac{1570}{2}\ \boxed{785}$

18. According to Newton's Law of Heating (Cooling), the temperature T of an object increases at a rate proportional to the difference between its temperature and that of the surrounding air. Let the constant of proportionality be k. Suppose a yam is put in an oven with temperature $T_s = 200°$ C.

a) If the yam is at 20° C when it is put in the oven, find the solution to the differential equation in terms of k.

b) Find the rate constant k using the fact that after 30 minutes the temperature of the yam is 120° C.

c) How long will it take for the yam to reach the temperature of 190° C?

Hints: the differential equation is

$$\frac{dT}{dt} = -k(T - T_s)$$

See also our CSV Textbook, chapter 11.5, page 547 problem 3 and Barron's Set 9, pages 326-327 questions 41, 42, 47 and 48.

a) $\frac{dT}{dt} = -k(T - 200)$

Let $y = T - 200$

$\frac{dy}{dt} = -ky$

$y = y_0 e^{-kt}$

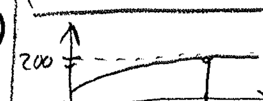
$y_0 = T_0 - 200$

$= 20 - 200$

$y = -180e^{-kt}$

$T - 200 = -180e^{-kt}$

$\boxed{T = 200 - 180e^{-kt}}$



b) $120 = 200 - 180e^{-30k}$

$180e^{-30k} = 80$

$e^{-30k} = \frac{4}{9}$

$k = -\ln(4/9)/30$

$\boxed{k = 0.027031/\text{MIN}}$

c) $190 = 200 - 180e^{-kt}$

$t = \ln(1/18)/k$

$\boxed{= 106.928\text{MIN}}$