TABLE OF DERIVATIVES AND INDEFINITE INTEGRALS: ANALYTICAL FORMULAS

CONSTANTS: a, C, k, n

FUNCTIONS: $u = u(x)$ $v = v(x)$

$du = u'(x)\,dx$

I. $\int f(x)\,dx = F(x) + C$ if $\dfrac{dF(x)}{dx} = f(x)$.

1. $\dfrac{da}{dx} = 0$

2. $\dfrac{d}{dx}au = a\dfrac{du}{dx}$

4. $\dfrac{d}{dx}(u+v) = \dfrac{d}{dx}u + \dfrac{d}{dx}v$; $\dfrac{d}{dx}(u-v) = \dfrac{d}{dx}u - \dfrac{d}{dx}v$

5. $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx} = \dfrac{du}{dx}v + u\dfrac{dv}{dx}$

6. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} = \dfrac{\dfrac{du}{dx}v - u\dfrac{dv}{dx}}{v^2}$ $(v \neq 0)$

3. $\dfrac{d}{dx}u^a = au^{a-1}\dfrac{du}{dx}$ (the power rule); $\dfrac{d}{dx}x^n = nx^{n-1}$

13. $\dfrac{d}{dx}\ln u = \dfrac{1}{u}\dfrac{du}{dx}$

A.* $\dfrac{d}{du}\operatorname{Log}_a(u) = \dfrac{1}{u\ln(a)}\dfrac{du}{dx}$

7. $\dfrac{d}{dx}\sin u = \cos u\dfrac{du}{dx}$

8. $\dfrac{d}{dx}\cos u = -\sin u\dfrac{du}{dx}$

B. If $y = f(u)$ and $u = g(x)$

B1. $(f(g(x)))' = f'(g(x)) \cdot g'(x) =$

B2. $= f'(u) \cdot g'(x) =$

B3, B4. $\dfrac{d}{du}f(u) \cdot \dfrac{du}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$. (chain rule)

C. $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$

D. $\sec(x) = 1/\cos(x)$

E. $\csc(x) = 1/\sin(x)$

J. $\int du = u + C$ $\int a\,du = au + C$

I1. $\int kf(x)\,dx = k\int f(x)\,dx$ $(k \neq 0)$

I2. $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

K.*** $\int u\,dv = uv - \int v\,du$ (by parts)

L. $\int u\,du = \dfrac{u^2}{2} + C$

I3. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C$ $(n \neq -1)$

I4. $\int \dfrac{du}{u} = \ln|u| + C$

M. $\int \cos(ax) = \dfrac{\sin(ax)}{a} + C$

I5. $\int \cos u\,du = \sin u + C$

I6. $\int \sin u\,du = -\cos u + C$

N1. $\int f(u)\,du = \int f(u)\dfrac{du}{dx} \cdot dx$

N2. $= \int f(u) \cdot u'(x)\,dx$ (substitution)

I7.** $\int \tan u\,du = \ln|\sec u| + C$

or $-\ln|\cos u| + C$

I8.** $\int \cot u\,du = \ln|\sin u| + C$

or $-\ln|\csc u| + C$

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MEMORIZE ALL FORMULAS EXCEPT THOSE MARKED ** OR ***.
TRIGONOMETRIC IDENTITIES CAN HELP IN FINDING FORMULAS.

* = MEMORIZE BY KNOWING HOW IT WAS FOUND

** = DON'T MEMORIZE BUT KNOW HOW TO FIND IT.

*** = NOT REQUIRED

FROM BARRON'S (8th ed., 2005).
NUMBERING FROM Ch. 3B AND Ch. 5B.

9. $\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$

10. $\frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}$

11. $\frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}$

12. $\frac{d}{dx} \csc u = -\csc u \cot u \frac{du}{dx}$

14. $\frac{d}{dx} e^u = e^u \frac{du}{dx}$

15.* $\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$

F1. $\frac{d}{dx} f^{-1}(x) = \dfrac{1}{\frac{d}{dx} f(f^{-1}(x))}$
at $(x, f^{-1}(x))$.

F2. If $u(x) = f^{-1}(x)$

$\frac{du}{dx} = \dfrac{1}{\frac{d}{du} f(u)}$ (inverses)

16. $\frac{d}{dx} \sin^{-1} u = \frac{d}{dx} \arcsin u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad (-1 < u < 1)$

17.* $\frac{d}{dx} \cos^{-1} u = \frac{d}{dx} \arccos u = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad (-1 < u < 1)$

18. $\frac{d}{dx} \tan^{-1} u = \frac{d}{dx} \arctan u = \frac{1}{1+u^2} \frac{du}{dx}$

19.* $\frac{d}{dx} \cot^{-1} u = \frac{d}{dx} \operatorname{arccot} u = -\frac{1}{1+u^2} \frac{du}{dx}$

20. $\frac{d}{dx} \sec^{-1} u = \frac{d}{dx} \operatorname{arcsec} u = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx} \quad (|u| > 1)$

21.* $\frac{d}{dx} \csc^{-1} u = \frac{d}{dx} \operatorname{arccsc} u = -\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx} \quad (|u| > 1)$

G. $\sin(2x) = 2\sin(x)\cos(x)$

H1. $\cos(2x) = \cos^2(x) - \sin^2(x) =$

H2* $= 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

I9. $\int \sec^2 u \, du = \tan u + C$

I10. $\int \csc^2 u \, du = -\cot u + C$

I11. $\int \sec u \tan u \, du = \sec u + C$

I12. $\int \csc u \cot u \, du = -\csc u + C$

I13.*** $\int \sec u \, du = \ln |\sec u + \tan u| + C$

I14.*** $\int \csc u \, du = \ln |\csc u - \cot u| + C$

I15. $\int e^u \, du = e^u + C$

I16.* $\int a^u \, du = \frac{a^u}{\ln a} + C \quad (a > 0, a \neq 1)$

O. $\cos^{-1}(x) = \pi/2 - \sin^{-1}(x)$
P. $\cot^{-1}(x) = \pi/2 - \tan^{-1}(x)$
Q. $\sec^{-1}(x) = \cos^{-1}(1/x)$
R1. $\csc^{-1}(x) = \pi/2 - \sec^{-1}(x)$
R2. $= \sin^{-1}(1/x)$
S.* $\cot^{-1}(x) = \tan^{-1}(1/x)$ ($x > 0$ ONLY)

T. $\cos^2(x) + \sin^2(x) = 1$
U.* $1 + \tan^2(x) = \sec^2(x)$
V.* $1 + \cot^2(x) = \csc^2(x)$

I17.* $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$

or $\arcsin u + C$

I18.* $\int \frac{du}{1+u^2} = \tan^{-1} u + C$

or $\arctan u + C$

I19.* $\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1} |u| + C$

or $\operatorname{arcsec} |u| + C$

W.** $\int \sin^2(u)\, du = \frac{u}{2} - \frac{\sin(2u)}{4} + C$

X.** $\int \cos^2(u)\, du = \frac{u}{2} + \frac{\sin(2u)}{4} + C$