NAME = _____ Period = ____

TAKE-HOME QUESTIONS

YOU ARE NOT EXPECTED TO WORK THIS FAST YET.

Section I Part A — 24 of 28 Questions = USE 47 of 55 MINUTES.
NOTE: THIS TEST IS MORE DIFFICULT THAN THE AP.

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = (2x^2 + 1)^4$, then $\frac{dy}{dx} =$

   (A) $16x^3$

   (B) $4(2x^2 + 1)^3$

   (C) $4x(2x^2 + 1)^3$

   (D) $16(2x^2 + 1)^3$

   (E) $16x(2x^2 + 1)^3$

Answer ☐

---

2. $\int_0^2 (2x^3 + 3)\, dx =$

   (A) 8

   (B) 11

   (C) 14

   (D) 20

   (E) 24

Answer ☐

---

3. If $\frac{dy}{dx} = 2xy$, then $\frac{d^2y}{dx^2} =$

(A) $2y$

(B) $2x + 2y$

(C) $2x + 4y$

(D) $2x^2y + 2y$

(E) $4x^2y + 2y$

Answer

4. The graph of $y = 2x^3 + 24x - 18$ is

(A) increasing for all x

(B) decreasing for all x

(C) only increasing for all x such that $|x| > 2$

(D) only increasing for all x such that $|x| < 2$

(E) only decreasing for all x such that $x < -2$

Answer

5. If $f = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x + 1, & x > 1 \end{cases}$, then $f'(1)$ is

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 3 (E) nonexistent

Answer

6. What is the maximum value of the derivative of $f(x) = 3x^2 - x^3$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Answer

7. Let f be a differentiable function for all x. Which of the following must be true?

I. $\frac{d}{dx}\int_0^3 f(x)\,dx = f(x)$

II. $\int_3^x f'(x)\,dx = f(x)$

III. $\frac{d}{dx}\int_3^x f(x)\,dx = f(x)$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

Answer

8. If $\sin(xy) = x^2$, then $\frac{dy}{dx} =$

(A) $2x\sec(xy)$

(B) $\frac{\sec(xy)}{x^2}$

(C) $2x\sec(xy) - y$

(D) $\frac{2x\sec(xy)}{y}$

(E) $\frac{2x\sec(xy) - y}{x}$

Answer

9. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 3t^3 - 18t^2 + 24t$. At which time t is its average velocity zero?

(A) Never

(B) 0 only

(C) 2 only

(D) 2 and 4 only

(E) 0, 2, and 4

Answer

10. How many points of inflection does the graph of $y = 2x^6 + 9x^5 + 10x^4 - x + 2$ have?

(A) None

(B) One

(C) Two

(D) Three

(E) Four

Answer

11. FIND $\int_0^4 (x^2 - 6x + 9)\,dx$ approximated by FOUR rectangles of equal width on the x-axis, if the function is evaluated at the right endpoints of each subinterval.

(A) 14

(B) 10

(C) 6

(D) 5

(E) 4

Answer

12. What is the 20th derivative of $y = \sin(2x)$?

(A) $-2^{20}\sin(2x)$

(B) $2^{20}\sin(2x)$

(C) $-12^{19}\cos(2x)$

(D) $2^{20}\cos(2x)$

(E) $2^{21}\cos(2x)$

Answer

13. What is the equation of the line tangent to the graph of $f(x) = 7x - x^2$ at the point where $f'(x) = 3$?

(A) $y = 5x - 10$

(B) $y = 3x + 4$

(C) $y = 3x + 8$

(D) $y = 3x - 10$

(E) $y = 3x - 16$

Answer

14. Suppose that $f(x)$ is a twice-differentiable function on the closed interval $[a, b]$. If there is a number c, $a < c < b$, for which $f'(c) = 0$, which of the following must be true?

I. $f(a) = f(b)$

II. f has a relative extremum at $x = c$.

III. f has a point of inflection at $x = c$.

(A) None

(B) I only

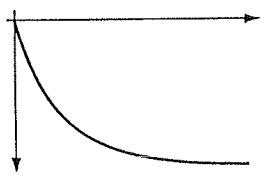
(C) II only

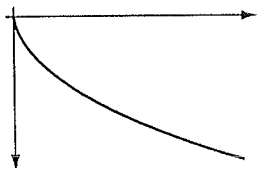
(D) I and II

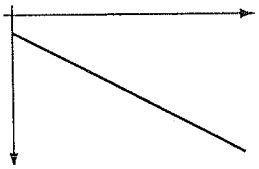
(E) II and III

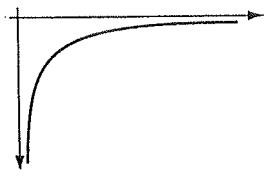
Answer

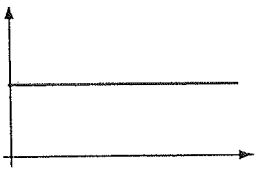
15. A sky diver has a negative velocity while falling from an airplane. Before the sky diver opens the parachute, her velocity decreases quickly and then levels off due to air resistance. Which graph approximates the acceleration of the sky diver?

(A) 

(B) 

(C) 

(D) 

(E) 

Answer

16. If $f(x) = x\sqrt[3]{x}$, then $f'(x) =$

(A) $4x^3$

(B) $\frac{3}{7}x^{\frac{7}{3}}$

(C) $\frac{4}{3}x^{\frac{1}{3}}$

(D) $\frac{1}{3}x^{\frac{1}{3}}$

(E) $\frac{1}{3}x^{-\frac{2}{3}}$

Answer

17. If $k > 0$ and $\int_k^6 \frac{dx}{x+2} = \ln k$, then $k =$

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

Answer

SKIP PART A: 18

(NOTE: IS WAS SKIPPED)

19. Let $f(x)$ be a continuous and differentiable function on the interval $0 \leq x \leq 1$, and let $g(x) = f(3x)$. The table below gives values of $f'(x)$, the derivative of $f(x)$. What is the value of $g'(0.1)$?

x	0.1	0.2	0.3	0.4	0.5	0.6
$f'(x)$	1.01	1.041	1.096	1.179	1.298	1.486

(A) 1.010

(B) 1.096

(C) 1.486

(D) 3.030

(E) 3.288

Answer

20. For what value of k will $\frac{8x+k}{x^2}$ have a relative maximum at $x = 4$?

(A) -32

(B) -16

(C) 0

(D) 16

(E) 32

Answer

21. $\lim_{h \to 0} \frac{2(x+h)^5 - 5(x+h)^3 - 2x^5 + 5x^3}{h}$ is

(A) 0

(B) $10x^3 - 15x$

(C) $10x^4 + 15x^2$

(D) $10x^4 - 15x^2$

(E) $-10x^4 + 15x^2$

Answer

22. If $\int_2^8 f(x)\,dx = -10$ and $\int_2^4 f(x)\,dx = 6$, then $\int_8^4 f(x)\,dx =$

(A) -16

(B) -6

(C) -4

(D) 4

(E) 16

Answer

23. If the graph of $y = x^3 + ax^2 + bx - 8$ has a point of inflection at $(2, 0)$, what is the value of b?

(A) 0

(B) 4

(C) 8

(D) 12

(E) The value of b cannot be determined from the given information.

Answer

24. If $f(x) = x^{-\frac{1}{3}}$, what is the derivative of the inverse of $f(x)$?

(A) $x^{\frac{1}{3}}$

(B) $-\frac{1}{3}x^{-\frac{4}{3}}$

(C) $\frac{1}{3}x^{-\frac{2}{3}}$

(D) $-3x^{-2}$

(E) $-3x^{-4}$

Answer

25. If f is a continuous function on the closed interval $[a, b]$, which of the following statements are NOT necessarily true?

I. f has a minimum on $[a, b]$.

II. f has a maximum on $[a, b]$.

III. $f'(c) = 0$ for some number c, $a < c < b$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

Answer

PART IA
PROBLEMS 26, 27, 28 SKIPPED

YOU ARE NOT EXPECTED TO WORK THIS FAST YET:

Section I Part B 11 of 17 Questions: Use 32 of 50 MINUTES

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may be used on this part of the exam.

In this test:

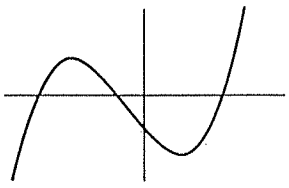
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices, the number that best approximates the exact numerical value.

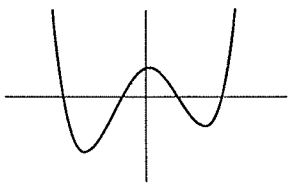
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

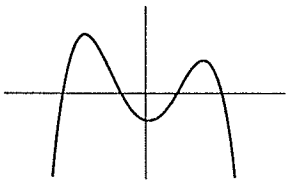
PART I B QUESTIONS 29 AND 30 SKIPPED

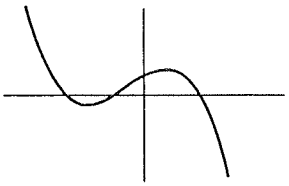
NOTE 31.

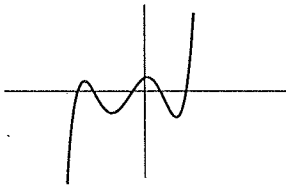
31. If $f'(x) = (x-a)(x-b)(x-c)$ and $a < b < c$, then which of the following could be the graph of $f(x)$?

(A) 

(B) 

(C) 

(D) 

(E) 

Answer

32. The equation of the horizontal asymptote for the graph of $y = \dfrac{2 - e^{\frac{1}{x}}}{2 + e^{\frac{1}{x}}}$ is

(A) $y = -1$ (B) $y = -\dfrac{1}{2}$ (C) $y = \dfrac{1}{3}$ (D) $y = \dfrac{1}{2}$ (E) $y = 1$

Answer

33. Let f be a function which is continuous on $[2, 10]$ and whose derivative is given by $f'(x) = \dfrac{\cos x}{\ln(x+1)}$. Which of the following are true about $f(x)$ on the interval $[2, 10]$?

I. $f(x)$ is monotonic.

II. $f(x)$ has a relative minimum.

III. $f(x)$ has three points of inflection.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

Answer

PART IB 34, 35 SKIPPED

NOTE: 36

36. The expression $\frac{1}{4}\left(\sqrt{1}+2\sqrt{5/4}+2\sqrt{2}+2\sqrt{13/4}+\sqrt{5}\right)$ is the trapezoidal approximation for which of the following definite integrals?

(A) $\int_1^3 \sqrt{x}\,dx$

(B) $\int_1^5 \sqrt{x}\,dx$

(C) $\int_0^4 \sqrt{x^2+1}\,dx$

(D) $\int_0^2 \sqrt{x^2+1}\,dx$

(E) $\int_{-1}^2 \sqrt{x^2+1}\,dx$

Answer

37. The average value of the function $f(x) = e^{-x}\sin x$ on the closed interval $[1, \pi]$ is

(A) 0.129

(B) 0.145

(C) 0.155

(D) 0.276

(E) 0.310

Answer

38. The position of an object attached to a spring is given by $y(t) = \frac{1}{3}\sin(4t) - \frac{1}{8}\cos(4t)$ where t is time in seconds. How many times does the acceleration of the object change from negative to positive in the first 5 seconds?

(A) Three

(B) Four

(C) Five

(D) Six

(E) Seven

PART IB
39, 40 SKIPPED

Answer

NOTE
41

41. Let f be the function given by $f(x) = x^2 \ln x$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 2?

(A) 1.305

(B) 1.548

(C) 2.000

(D) 2.548

(E) 4.773

Answer

42. The mass $m(t)$, in grams, of a tumor t weeks after it begins growing is given by $m(t) = \frac{te^t}{80}$. What is the average rate of change, in grams per week, during the fifth week of growth?

(A) 2.730

(B) 3.412

(C) 6.189

(D) 6.546

(E) 11.131

Answer

43. Let $f(x)$ be a differentiable function defined for all real numbers. The table below gives the value of $f(x)$ and its derivative $f'(x)$ for several values of x.

x	−3	−2	−1	0	1	2	3
$f(x)$	8	5	0	1	0	5	8
$f'(x)$	−6	−4	−2	0	2	4	6

Which of the following statements are true?

I. At $x = 2$, the function is increasing.

II. There is a relative minimum in the interval $-1 \leq x \leq 1$, but not necessarily at $x = 0$.

III. There is a relative maximum in the interval $-1 \leq x \leq 1$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

Answer

44. A particle moves along the x-axis so that its position at any time $t > 0$ is given by $x(t) = t^3 + 22t + 3 - 6\cos(\pi t)$. For what value of t is the velocity negative?

(A) $t = \frac{1}{2}$

(B) $t = 1$

(C) $t = \frac{3}{2}$

(D) $t = 2$

(E) The velocity is never negative.

Answer

45. The closed interval $[0, \pi]$ is partitioned into n equal subdivisions each of length $\Delta x = \frac{\pi}{n}$ by the numbers $x_0, x_1, x_2 \ldots, x_{n-1}, x_n$, with $0 = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = \pi$.

The $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \cos(x_i)\Delta x$ is

(A) -2

(B) -1

(C) 1

(D) 2

(E) π

Answer