1/16/08

Calculus AB Sample Free-Response Questions 2006-2007 (AP)

Sheet # 530

Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

Question 3

A particle moves along the y-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

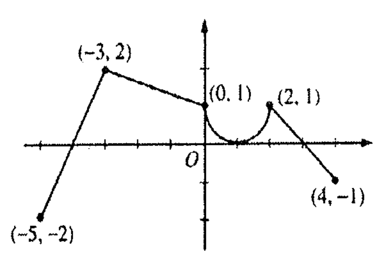
(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

Question 5

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

1.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars

$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(b) $F'(7) = -1.872$ or -1.873
Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.

1 : answer with reason

(c) $\frac{1}{5}\int_0^{15} F(t)\,dt = 81.899$ cars/min

$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(d) $\dfrac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2

1 : answer

Units of cars/min in (c) and cars/min^2 in (d)

1 : units in (c) and (d)

3.

(a) $a(2) = v'(2) = -0.132$ or -0.133

1 : answer

(b) $v(2) = -0.436$
Speed is increasing since $a(2) < 0$ and $v(2) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $\tan^{-1}\left(e^t\right) = 1$
$t = \ln(\tan(1)) = 0.443$ is the only critical value for y.

$v(t) > 0$ for $0 < t < \ln(\tan(1))$
$v(t) < 0$ for $t > \ln(\tan(1))$

$y(t)$ has an absolute maximum at $t = 0.443$.

$3: \begin{cases} 1: \text{sets } v(t) = 0 \\ 1: \text{identifies } t = 0.443 \text{ as a candidate} \\ 1: \text{justifies absolute maximum} \end{cases}$

(d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361
The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.

$4: \begin{cases} 1: \int_0^2 v(t)\,dt \\ 1: \text{handles initial condition} \\ 1: \text{value of } y(2) \\ 1: \text{answer with reason} \end{cases}$

5.

(a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$
$g'(0) = f(0) = 1$

$2: \begin{cases} 1: g(0) \\ 1: g'(0) \end{cases}$

(b) g has a relative maximum at $x = 3$.
This is the only x-value where $g' = f$ changes from positive to negative.

$2: \begin{cases} 1: x = 3 \\ 1: \text{justification} \end{cases}$

(c) The only x-value where f changes from negative to positive is $x = -4$. The other candidates for the location of the absolute minimum value are the endpoints.

$g(-5) = 0$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -1$

$g(4) = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) = \frac{13 - \pi}{2}$

So the absolute minimum value of g is -1.

$3: \begin{cases} 1: \text{identifies } x = -4 \text{ as a candidate} \\ 1: g(-4) = -1 \\ 1: \text{justification and answer} \end{cases}$

(d) $x = -3, 1, 2$

2 : correct values
$\langle -1 \rangle$ each missing or extra value