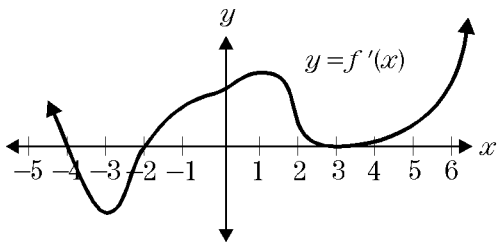
Sheet #453 - Ch.4.1-4.5 Review

Name _____ Date _____

Note: Make sure you can do each question with or without multiple choices given. Work longer questions at end early in your review. Vocabulary for extrema: relative=local, absolute=global. Critical number is x value of a critical point. TURN IN ALL WORK!

1. 

The figure shows the graph of $f'(x)$, the derivative of the function $f(x)$, for $-5 \le x \le 6$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 1$, and $x = 3$. It also has a vertical tangent line at $x = 2$.

a) Find all values of x, in the interior of the interval, for $-5 < x < 6$ at which f attains a local (relative) minimum. Justify your answer. [Also write down any end point minimum(s) if exist(s).]

b) Find all values of x, in the interior of the interval, for $-5 < x < 6$ at which f attains a local (relative) maximum. Justify your answer. [Also write down any end point maximum(s) if exist(s).]

c) Find all values of x, for $-5 \le x \le 6$ at which $f''(x) < 0$. Justify your answer.

d) At what value of x, for $-5 \le x \le 6$ does f attain its global (absolute) maximum? Justify your answer. [Also write down the global (absolute) minimum.]

2. Let $f(x) = x^2(x-3)$. Over what interval is the function increasing?

a) $0 < x < 2$

b) $0 < x < 3$

c) $0 < x < \infty$

d) $-\infty < x < 0$ and $x > 2$

e) $-\infty < x < \infty$

3. Given $f(x) = (1 + \ln x)^3$ where $x \ge 0$, find the points of inflection on the graph of $f(x)$.

a) $(e, 8)$ only b) $(\frac{1}{e}, 8)$ only c) $(e, 8)$, $(\frac{1}{e}, 8)$

d) $(e, 8)$, $(\frac{1}{e}, 0)$ e) $(0, 8)$, $(\frac{1}{e}, e)$

4. Given that $f'(x) = 2^{\sin x} - 3^{\cos x}$ for $0 < x < 5$ then f has a local max at $x \approx$

a) 4.150 b) 1.008 c) 2.048 d) 1.247 e) 3.158

5. Given that $f'(x) = xe^x - \sin x$ for $-4 < x < 0$ then f has an inflection point for $x \approx$

a) -3.999 b) -0.933 c) -2.467

d) -3.000 e) -1.699

6. Let $f(x) = x^3 - x^2 + 3$. Determine the critical numbers.

a) 0, $\frac{2}{3}$ b) 1, 3 c) 0, 3 d) $\frac{2}{3}$, 3 e) 1, $\frac{2}{3}$

7. Find all critical numbers of $f(x) = x\sqrt{2x+1}$.

a) 0, $-\frac{1}{3}$, and $-\frac{1}{2}$ b) $\frac{1}{2}$ and $\frac{1}{3}$

c) $-\frac{1}{3}$ and $-\frac{1}{2}$ d) 0 and $-\frac{1}{2}$

e) $-\frac{1}{3}$ only

8. Given $y = 3x^3 + 5x^2 + x + 24$, find the x-value of the midpoint between the two critical points.

a) $-\frac{11}{9}$ b) $-\frac{5}{9}$ c) $-\frac{1}{9}$ d) $\frac{1}{9}$ e) $\frac{5}{9}$

9. Find the x-coordinate of the relative maximum or relative minimum point of the function $f(x) = x^{1/3}(9-x)$.

a) $-\frac{1}{4}$ max b) $\frac{1}{4}$ min c) $-\frac{9}{4}$ min

d) $\frac{9}{4}$ max e) 9 max

10. Given a function is defined by $f(x) = \dfrac{x}{x^2+1}$. Find all relative maximum and/or relative minimum points.

a) $(1, \frac{1}{2})$ min only

b) $(-1, -\frac{1}{2})$ max only

c) $(1, \frac{1}{2})$ max; $(1, -\frac{1}{2})$ min

d) $(1, \frac{1}{2})$ min; $(-1, -\frac{1}{2})$ max

e) $(1, \frac{1}{2})$ max; $(-1, -\frac{1}{2})$ min

11. Note: y values are sought. Let $f(x) = 1 - \sin x - (\cos x)^2$ for $0 \le x \le \frac{3\pi}{2}$. Find the absolute maximum of the function f.

a) 1 b) 2 c) $-\frac{1}{4}$ d) 4 e) 1.5

12. Find the absolute maximum and absolute minimum of f on $(-1, 2]$.

$$f(x) = \frac{x^3 - x^2 - 3x - 1}{x+1}$$

a) Max: $(-1, 2)$, Min: $(2, -1)$

b) Max: None, Min: $(1, -2)$

c) Max: $(-1, 2)$, Min: $(1, -2)$

d) Max: None, Min: $(2, -1)$

e) Max: None, Min: $(0, -1)$

13. Note: y values are sought. Find the local maximum and the local minimum of

$$f(x) = \begin{cases} x^2 - 4 & \text{for } x \le 3, \\ -2x + 11 & \text{for } x > 3 \end{cases}$$

14. Find the point on the curve $f(x) = x^2 + 1$ that is nearest to the the point $B(3,1)$.

a) $(1,2)$ b) $(2,5)$ c) $(0,1)$ d) $(\frac{1}{3}, \frac{2}{3})$ e) $(5,2)$

15. Note: distinguish definitions of width, radius, diameter etc.

If the radius of the circle shown is 6, then what is the area of the largest rectangle that can be inscribed in the circle?

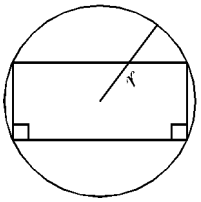
a) 36 units^2

b) 72 units^2

c) 324 units^2

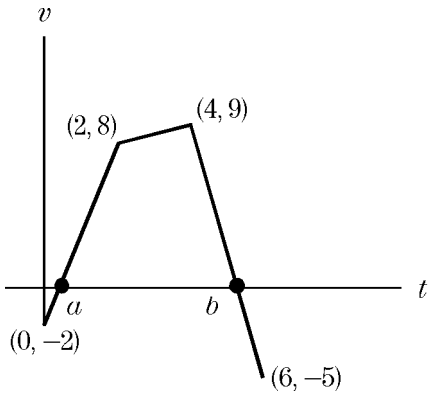
d) $18\sqrt{2}$ units^2

e) $9\sqrt{2}$ units^2



16. The graph shows the velocity of an object that is moving along a straight line for t on $[0,6]$.

At what time(s) t does the object reverse direction?



a) 2 and 4 b) a and b c) 4 only

d) 5 only e) a only

17. The graph shows the velocity of an object that is moving along a straight line for t on $[0,6]$.

What is the maximum speed of the object?

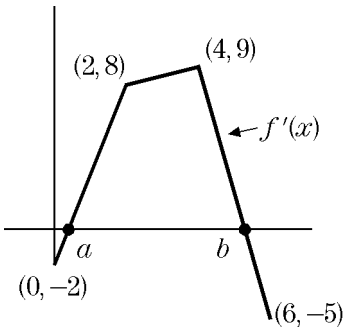
a) 6 units/sec

b) 4 units/sec

c) 9 units/sec

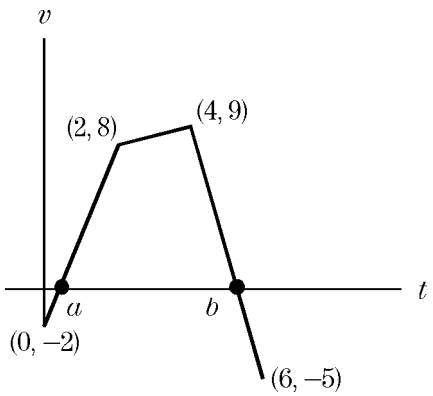
d) 5 units/sec

e) 8 units/sec



18. The graph shows the velocity of an object that is moving along a straight line for t on $[0,6]$.

The object is furthest to the right when $t =$ _____.



a) 0 b) b c) a d) 6 e) 4

19. A particle starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = (2t^2 - 5t + 3)(t - 1)^2$. Find the value of t when the acceleration is zero and the particle is moving.

a) 1 b) 2 c) 0 d) $\frac{5}{4}$ e) $\frac{11}{8}$

20. A right circular cylinder is to be designed to hold 22 cubic inches of a soft drink. The cost for the material for the top and bottom of the can is twice the cost for the material of the sides. Let r represent the radius and h the height of the cylinder.

a) Write the equation for the surface area, SA, in r and h.

b) Write the cost function C.

c) Write the cost function as a function of one variable, r.

d) Find the radius that minimizes cost.

21. An open top box is to be made from a 3-foot by 4-foot rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made in this manner.

a) 4.01 cubic feet b) 3.92 cubic feet

c) 3.03 cubic feet d) 2.08 cubic feet

e) None of these

22. Consider a company with the revenue function $R = 4x$, and the cost function $C = \frac{4}{3}x^3 - 6x^2 + 9.5x + 1$ as a function of x, the number of units produced. Assume that every unit produced is sold. *Multipliers:* The functions R and C are given in tens of thousands of dollars and the variable x is given in hundreds of units. (Pay close attention to these multipliers in your answers!)

a) What is the price per unit?

b) What is the fixed cost?

c) *Non-calculus Marginal Cost:* What is the increase in cost from selling 100 units to selling 101 units?

d) *Calculus Marginal Cost:* Find the marginal cost at 100 units using rates of change. Round to three decimals.

e) If 100 units are being produced and sold, is it better to produce and sell more units or fewer units in order to increase profit? Why? Explain in terms of marginal cost and marginal revenue.

f) Use a graphing calculator to find the number of units that should be sold to maximize profit. Give both the number of units and the maximum profit. Use three decimals.

g) Use calculus to write down the algebraic equation to be solved to maximize profit. Solve the equation *exactly*. (No decimals.)

h) Use the second derivative test to algebraically justify that you have a maximum.

i) Give a final answer for the maximum profit and the number of units that maximizes the profit. Make sure to give answers that are appropriate and make sense in this situation.