

**Optimizing the Volume of an Open-top Box.**

**Materials:** Pieces of paper (11 inches by 8.5 inches), tape, rulers .

In this investigation, we study how to make a box with as large a volume as possible.

a) Fold a piece of paper to make a box with as large a volume as possible. The piece of paper should be folded into an open-top box after cutting out equally-sized squares from the corners. No calculators allowed!

b) Tape the box together and measure its dimensions *in millimeters*. Compute the volume and write it on the box.

c) Gather in small groups to compare your boxes and to discuss your findings. Write down your observations.

d) Give your box to the teacher who will line up all boxes in the front of the classroom in order of increasing volume. Be prepared to discuss what worked and what did not work.

e) Find an algebraic formula for the volume as a function of height ( $x$ ). Be sure to state the boundary conditions on the height ( $x$ ) so that the function makes sense in this context.

f) Use your graphing calculator to find the maximum volume and the three dimensions of the box with the maximum volume.

g) Use algebraic methods to verify your results using calculus.

h) Compare the results from parts f and g with the actual boxes that were made by the whole class. How did your box compare? Write down your observations.

Further exploration: What would you do if you were asked to make a closed-top box from a piece of paper the same size? Find the dimensions of the box with the maximum volume. Could you explain the result in a simple way?