SHEET #452

NAME: _____

Period: ____

π^e Versus e^π

Students conjecture that $\pi^e > e^\pi$ because "the base is bigger."

What really determines which is bigger, a^b or b^a?

a) Compare $2.8^{2.9}$ with $2.9^{2.8}$; could you tell in advance which is bigger?

b) Compare $2.6^{2.7}$ with $2.7^{2.6}$; which is bigger now?

c) Graph $y = x^{1/x}$ on your grapher. Use the graph to determine where the function is increasing, decreasing, and where it has its maximum. MAKE A SKETCH.

d) Relate the graph, and your answers in part c), to your results from parts a) and b).

Hint: $a^{1/a} > b^{1/b} \Rightarrow a^b > b^a$ if $a,\ b > 0$ (Why?)

e) Now you should be able to determine without calculator which is bigger, $2.5^{2.6}$ or $2.6^{2.5}$.

f) Note that $y = x^{1/x} \Rightarrow \ln(y) = \frac{1}{x}\ln(x)$. Using this last equation take derivatives and solve for y'.

Use the derivative to verify your answers in c) above (Hint: use the first derivative test).