

Problems **8-1**

In words, give a practical interpretation of the functions in Problems 17–20.

- 17. $f(h(t))$, where $A = f(r)$ is the area of a circle of radius r and $r = h(t)$ is the radius of the circle at time t .
- 18. $k(g(t))$, where $L = k(H)$ is the length of a steel bar at temperature H and $H = g(t)$ is temperature at time t .
- 19. $R(Y(q))$, where R gives a farmer's revenue as a function of corn yield per acre, and Y gives the corn yield as a function of the quantity, q , of fertilizer.
- 20. $t(f(H))$, where $t(v)$ is the time of a trip at velocity v , and $v = f(H)$ is velocity at temperature H .
- 21. Complete Table 8.8 given that:

- f is symmetric about the y -axis (even).
- g is symmetric about the origin (odd).
- h is the composition of g with f , that is, $h(x) = g(f(x))$.

Table 8.8

x	-3	-2	-1	0	1	2	3
$f(x)$	0	2	2	0			
$g(x)$	0	2	2	0			
$h(x)$							

22. Complete Table 8.9 given that $h(x) = f(g(x))$.

Table 8.9

x	$f(x)$	$g(x)$	$h(x)$
0	1	2	5
1	9	0	
2		1	

23. Using Figure 8.2, estimate the following:
 (a) $f(g(2))$ (b) $g(f(2))$ (c) $f(f(3))$ (d) $g(g(3))$

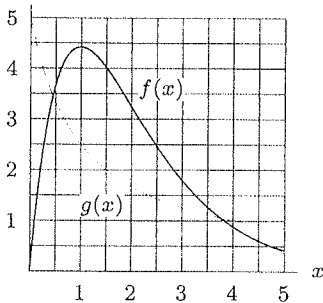


Figure 8.2

In Problems 24–27, use the information from Figures 8.3, and 8.4 to graph the functions.

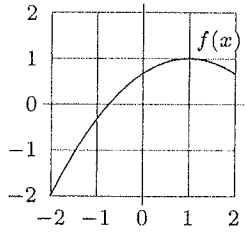


Figure 8.3

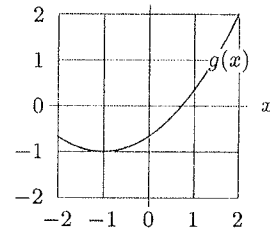


Figure 8.4

24. $f(g(x))$ 25. $g(f(x))$ 26. $f(f(x))$ 27. $g(g(x))$

28. Complete Tables Table 8.10, Table 8.11, and Table 8.12 given that $h(x) = g(f(x))$. Assume that different values of x lead to different values of $f(x)$.

Table 8.10

x	$f(x)$
-2	4
-1	
0	
1	5
2	1

Table 8.11

x	$g(x)$
1	
2	1
3	2
4	0
5	-1

Table 8.12

x	$h(x)$
-2	
-1	1
0	2
1	
2	-2

29. Use Figure 8.5 to calculate the following:

- (a) $f(f(1))$ (b) $g(g(1))$
- (c) $f(g(2))$ (d) $g(f(2))$

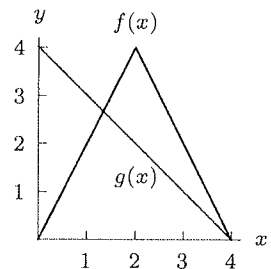


Figure 8.5

30. Use Figure 8.5 to find all solutions to the equations:

(a) $f(g(x)) = 0$ (b) $g(f(x)) = 0$

31. Let $f(x)$ and $g(x)$ be the functions in Figure 8.5.

- (a) Graph the functions $f(g(x))$ and $g(f(x))$.
- (b) On what interval(s) is $f(g(x))$ increasing?
- (c) On what interval(s) is $g(f(x))$ increasing?

32. Find $f(f(1))$ for

$$f(x) = \begin{cases} 2 & \text{if } x \leq 0 \\ 3x + 1 & \text{if } 0 < x < 2 \\ x^2 - 3 & \text{if } x \geq 2 \end{cases}$$

33. Using your knowledge of the absolute value function, explain in a few sentences the relationship between the graph of $y = |\sin x|$ and the graph of $y = \sin x$.

34. Graph the following functions for $-2\pi \leq x \leq 2\pi$.

(a) $f(x) = \sin x$ (b) $g(x) = |\sin x|$
 (c) $h(x) = \sin |x|$ (d) $i(x) = |\sin |x||$

(e) Do any two of these functions have identical graphs? If so, explain why this makes sense.

- 35. (a) Using what you know about the graph of $y = \sin x$, make predictions about the graph of $y = (\sin x)^2$.
- (b) Using a computer or a graphing calculator, graph $g(x) = (\sin x)^2$, $h(x) = (\cos x)^2$ and $f(x) = g(x) + h(x)$, on the interval $-2\pi \leq x \leq 2\pi$. What do you observe about the graph of $f(x)$?

In Problems 36–39, find a simplified formula for the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

36. $f(x) = x^2 + x$ 37. $f(x) = \sqrt{x}$

38. $f(x) = \frac{1}{x}$ 39. $f(x) = 2^x$

40. Let $f(x) = 1/x$. For n a positive integer, define $f_n(x)$ as the composition of f with itself n times. For example, $f_2(x) = f(f(x))$ and $f_3(x) = f(f(f(x)))$.

- (a) Evaluate $f_7(2)$.
- (b) Evaluate $f_{23}(f_{22}(5))$.

Decompose the functions in Problems 41–48 into two new functions, u and v , where v is the inside function, $u(x) \neq x$, and $v(x) \neq x$.

41. $f(x) = \sqrt{3 - 5x}$

42. $F(x) = (2x + 5)^3$

43. $h(x) = x^4 + x^2$

44. $j(x) = 1 - \sqrt{x}$

45. $l(x) = 2 + \frac{1}{x}$

46. $G(x) = \frac{2}{1 + \sqrt{x}}$

47. $H(x) = 3^{2x-1}$

48. $J(x) = 8 - 2|x|$

For Problems 49–53, let $k(x) = x^2 + 2$ and $g(x) = x^2 + 3$. Find a possible formula for the function named.

49. $h(x)$ if $h(k(x)) = (x^2 + 2)^3$.

50. $j(x)$ if $k(j(t)) = \left(\frac{1}{t}\right)^2 + 2$.

51. $f(x)$ if $f(k(v)) = \frac{1}{v^2 + 2}$.

52. $h(x)$ if $h(g(x)) = \frac{1}{x^2 + 3} + 5x^2 + 15$.

53. $j(x)$ if $j(x) = g(g(x))$.

54. Suppose $p(x) = (1/x) + 1$ and $q(x) = x - 2$.

- (a) Let $r(x) = p(q(x))$. Find a formula for $r(x)$ and simplify it.
- (b) Write formulas for $s(x)$ and $t(x)$ such that $p(x) = s(t(x))$, where $s(x) \neq x$ and $t(x) \neq x$.
- (c) Let a be different from 0 and -1 . Find a simplified expression for $p(p(a))$.

55. If $s(x) = 5 + \frac{1}{x+5} + x$, $k(x) = x + 5$, and $s(x) = v(k(x))$, what is $v(x)$?

56. Suppose $u(v(x)) = \frac{1}{x^2 - 1}$ and $v(u(x)) = \frac{1}{(x - 1)^2}$. Find possible formulas for $u(x)$ and $v(x)$.

57. Assume that $f(x) = 3 \cdot 9^x$ and that $g(x) = 3^x$.

- (a) If $f(x) = h(g(x))$, find a formula for $h(x)$.
- (b) If $f(x) = g(j(x))$, find a formula for $j(x)$.

58. You have two money machines, both of which increase any money inserted into them. The first machine doubles your money. The second adds five dollars. The money that comes out is described by $d(x) = 2x$, in the first case, and $a(x) = x + 5$, in the second, where x is the number of dollars inserted. The machines can be hooked up so that the money coming out of one machine goes into the other. Find formulas for each of the two possible composition machines. Is one machine more profitable than the other?

Exercises and Problems for Section 8.2

Exercises

In Exercises 1–4, use a graph to decide whether or not the function is invertible.

1. $y = x^4 - 6x + 12$

2. $y = x^6 + 2x^2 - 10$

3. $y = \sqrt{x} + x$

4. $y = |x|$

In Exercises 5–8, check that the functions are inverses.

5. $f(x) = \frac{x}{4} - \frac{3}{2}$ and $g(t) = 4\left(t + \frac{3}{2}\right)$

6. $f(x) = 1 + 7x^3$ and $f^{-1}(x) = \sqrt[3]{\frac{x-1}{7}}$

7. $g(x) = 1 - \frac{1}{x-1}$ and $g^{-1}(x) = 1 + \frac{1}{1-x}$

8. $h(x) = \sqrt{2x}$ and $k(t) = \frac{t^2}{2}$, for $x, t \geq 0$

Find the inverses of the functions in Exercises 9–21.

9. $h(x) = 12x^3$

10. $h(x) = \frac{x}{2x+1}$

11. $k(x) = 3 \cdot e^{2x}$

12. $g(x) = e^{3x+1}$

13. $n(x) = \log(x-3)$

14. $h(x) = \ln(1-2x)$

15. $h(x) = \frac{\sqrt{x}}{\sqrt{x}+1}$

16. $f(x) = \frac{3+2x}{2-5x}$

17. $f(x) = \sqrt{\frac{4-7x}{4-x}}$

18. $f(x) = \frac{\sqrt{x}+3}{11-\sqrt{x}}$

19. $p(x) = 2 \ln\left(\frac{1}{x}\right)$

20. $s(x) = \frac{3}{2+\log x}$

21. $q(x) = \ln(x+3) - \ln(x-5)$

Problems

22. If $P = f(t)$ gives the population of a city, in thousands, as a function of time, t , in years, what does $f^{-1}(P)$ represent? What are its units?
23. If $C = f(q)$ gives the cost, in dollars, to manufacture q items, what does $f^{-1}(C)$ represent? What are its units?
24. If $t = g(v)$ represents the time in hours it takes to drive to the next town at velocity v mph, what does $g^{-1}(t)$ represent? What are its units?
25. Let $P = f(t) = 10e^{0.02t}$ give the population in millions at time t in years. Find and interpret $f^{-1}(P)$.
26. Let $C = f(q) = 200 + 0.1q$ give the cost in dollars to manufacture q kg of a chemical. Find and interpret $f^{-1}(C)$.
27. The noise level, N , of a sound in decibels is given by

$$N = f(I) = 10 \log\left(\frac{I}{I_0}\right),$$

where I is the intensity of the sound and I_0 is a constant. Find and interpret $f^{-1}(N)$.

Solve the equations in Problems 28–33 exactly. Use an inverse function when appropriate.

28. $7 \sin(3x) = 2$

29. $2^{x+5} = 3$

30. $x^{1.05} = 1.09$

31. $\ln(x+3) = 1.8$

32. $\frac{2x+3}{x+3} = 8$

33. $\sqrt{x+\sqrt{x}} = 3$

34. Values of f and g are in Table 8.18. Based on this table:

- (a) Is $f(x)$ invertible? If not, explain why; if so, construct a table of values of $f^{-1}(x)$ for all values of x for which $f^{-1}(x)$ is defined.
- (b) Answer the same question as in part (a) for $g(x)$.
- (c) Make a table of values for $h(x) = f(g(x))$, with $x = -3, -2, -1, 0, 1, 2, 3$.
- (d) Explain why you cannot define a function $j(x)$ by the formula $j(x) = g(f(x))$.

Table 8.18

x	-3	-2	-1	0	1	2	3
$f(x)$	9	7	6	-4	-5	-8	-9
$g(x)$	3	1	3	2	-3	-1	3

35. Figure 8.17 defines the function f . Rank the following quantities in order from least to greatest: $0, f(0), f^{-1}(0), 3, f(3), f^{-1}(3)$.

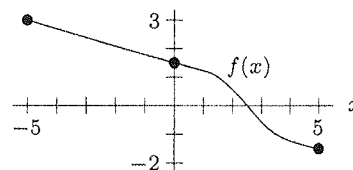
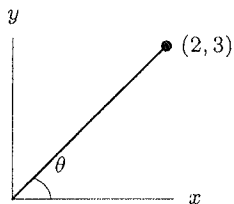


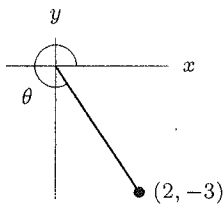
Figure 8.17

Section 7.5

1 0° to 90°



3 270° to 360°



- 5 IV
- 7 II
- 9 III
- 11 I
- 13 IV
- 15 $(-\sqrt{6}/2, -\sqrt{6}/2)$
- 17 $(-\sqrt{3}, 1)$
- 19 $(1, \pi)$
- 21 $(2, 5\pi/6)$
- 23 $H: x = 3, y = 0; r = 3, \theta = 0$
 $M: x = 0, y = 4; r = 4, \theta = \pi/2$
- 25 $H: x = 3\sqrt{2}/2, y = 3\sqrt{2}/2; r = 3, \theta = \pi/4$
 $M: x = 0, y = -4; r = 4, \theta = 3\pi/2$
- 27 $H: x \approx 2.898, y \approx -0.776; r = 3, \theta = 23\pi/12$
 $M: x = 0, y = -4; r = 4, \theta = 3\pi/2$
- 29 $\sqrt{8} \leq r \leq \sqrt{18}$ and $\pi/4 \leq \theta \leq \pi/2$
- 31 $0 \leq \theta \leq \pi/2$ and $1 \leq r \leq 2/\cos \theta$
- 33 n loops
- 35 An inner and outer loop
- 37 $0 \leq \theta \leq 2\pi$ and $3/16 \leq r \leq 1/2$
- 39 (a) $0 \leq \theta \leq \pi/4$ and $0 \leq r \leq 1$
(b) Two pieces:
 $0 \leq x \leq \sqrt{2}/2$ and $0 \leq y \leq x$;
 $\sqrt{2}/2 \leq x \leq 1$ and $0 \leq y \leq \sqrt{1-x^2}$

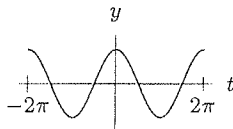
Section 7.6

- 1 $2e^{i\pi/2}$
- 3 $5e^{i4.069}$
- 5 $e^{i3\pi/2}$
- 7 $-3 - 4i$
- 9 $-5 + 12i$
- 11 $3 - 6i$

- 13 $\frac{\sqrt{3}}{2} + \frac{i}{2}$
- 15 $\sqrt{2} + i\sqrt{2}$
- 17 $\frac{\sqrt{3}}{2} + \frac{i}{2}$
- 19 $\sqrt{2}/2 + i\sqrt{2}/2$
- 21 $\sqrt{2} \cos \frac{\pi}{12} + i\sqrt{2} \sin \frac{\pi}{12}$
- 23 $2.426 + 4.062i$
- 25 $A_1 = 2 - i$
 $A_2 = -2 + 2i$
- 27 $p = -b$ and $q = \sqrt{c-b^2}$

Chapter 7 Review

- 1 (a) $a = 4; c = 2; B = 60^\circ$
(b) $A \approx 73.740^\circ; B \approx 16.260^\circ; b = 7$
- 3 $\phi = 53.130^\circ; \theta = 36.870^\circ$
- 5 5; $67.380^\circ; 22.620^\circ$
- 7 $\cos \theta$
- 9 $2 \cos \phi$
- 11 $-\sqrt{2}/2$
- 13 $(-\sqrt{6} - \sqrt{2})/4$
- 15 I
- 17 I
- 19 $(-0.832, 1.819)$
- 21 $(0, 0)$
- 23 Both are right
- 25 $\cos \theta = 3/\sqrt{14}$
- 27 $7\pi/6, 11\pi/6, \pi/2$
- 29 Appear to be same



- 35 $(\tan^2 x)(\sin^2 x) = \tan^2 x - \sin^2 x$
- 41 4 rolls
- 43 (a) OE
(b) OA
(c) DB
(d) OF
(e) OC
(f) GH

Ch. 7 Understanding

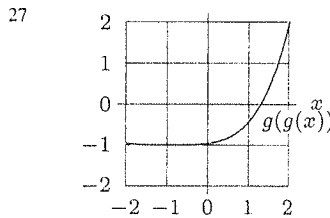
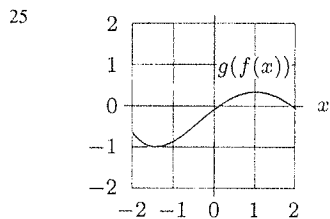
- 1 False
- 3 False
- 5 True
- 7 False
- 9 True
- 11 True
- 13 True
- 15 False
- 17 False
- 19 True
- 21 True

- 23 False
- 25 True
- 27 False
- 29 True
- 31 True
- 33 True
- 35 False
- 37 False
- 39 False
- 41 True
- 43 False
- 45 False
- 47 True

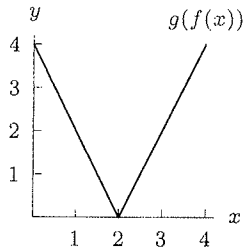
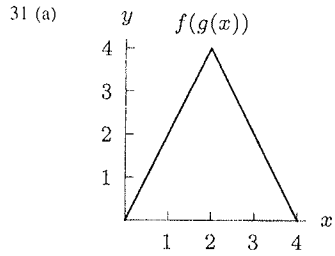
Section 8.1

More Answers
Appendix P.6

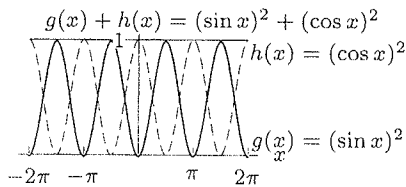
- 1 $r(0) = 4, r(1) = 5, r(2) = 2, r(3) = 0, r(4) = 3, r(5)$ undefined
- 3 $\pi/2, \pi/3, \pi/4, \pi/6, 0$
- 5 $(x^2 - 6x + 10)/(x^2 - 6x + 9)$
- 7 $x + 1$
- 9 $(x - 3)/(10 - 3x)$
- 11 $k(m(x)) = 1/(x - 1)^2$
- 13 $k(n(x)) = 4x^4/(x + 1)^2$
- 15 $(m(x))^2 = 1/(x^2 - 2x + 1)$
- 17 Area in terms of time
- 19 Revenue in terms of fertilizer
- 21 $f: 2, 2, 0$
 $g: -2, -2, 0$
 $h: 0, -2, -2, 0, -2, -2, 0$
- 23 (a) 3.7
(b) 1.2
(c) 3.6
(d) 2.2



- 29 (a) 4
(b) 1
(c) 4
(d) 0

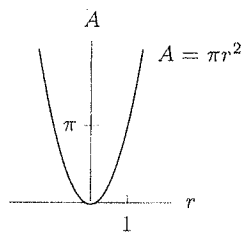


- (b) $0 < x < 2$
 (c) $2 < x < 4$
 35 (a) Periodic; Range: $0 \leq y \leq 1$
 (b) $(\sin x)^2 + (\cos x)^2 = 1$

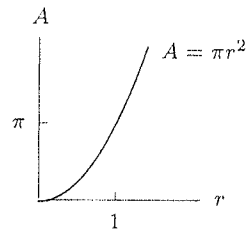


- 37 $(\sqrt{x+h} - \sqrt{x})/h$
 39 $(2^{x+h} - 2^x)/h$
 41 $u(x) = \sqrt{x}, v(x) = 3 - 5x$
 43 $u(x) = x^2 + x, v(x) = x^2$
 45 $u(x) = 2 + x, v(x) = 1/x$
 47 $u(x) = 3^x, v(x) = 2x - 1$
 49 $h(x) = x^3$
 51 $f(x) = 1/x$
 53 $j(x) = (x^2 + 3)^2 + 3$
 55 $v(x) = x + 1/x$
 57 (a) $h(x) = 3x^2$
 (b) $j(x) = 2x + 1$
 59 (a) $f(x) = 121.5x$
 $g(x) = 1.02638x$
 $h(x) = 0.00845x$
 (b) \$1000 trades for yen,
 then for 1026.675 euros

- 13 $n^{-1}(x) = 10^x + 3$
 15 $h^{-1}(x) = (x/(1-x))^2$
 17 $f^{-1}(x) = (4x^2 - 4)/(x^2 - 7)$
 19 $p^{-1}(x) = 1/e^{(x/2)}$
 21 $q^{-1}(x) = (3 + 5e^x)/(e^x - 1)$
 23 Quantity manufactured at cost C ; number of items
 25 $f^{-1}(P) = 50 \ln(P/10)$
 27 $f^{-1}(N) = I_0 10^{N/10}$
 29 $x = (\ln 3 / \ln 2) - 5$
 31 $x = e^{1.8} - 3$
 33 $x = (19 - \sqrt{37})/2$
 35 $f^{-1}(3) < f(3) < 0 < f(0) < f^{-1}(0) < 3$
 39 (a) $x = (1/3) \ln(1/2)$
 (b) $x \approx -0.382$
 (c) $x = (1/6) \ln 2$
 (d) $x = (1/4) \ln 2$
 41 $1 - t^2$
 43 (a) $A = \pi r^2$
 (b)

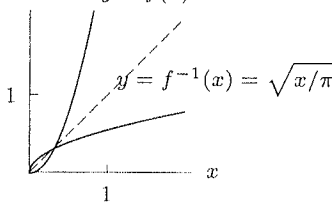


(c) $r \geq 0$



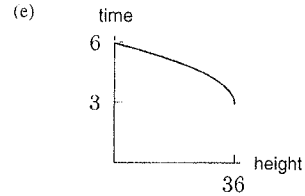
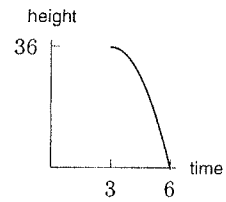
(d) $f^{-1}(A) = \sqrt{A/\pi}$

(e) $y = f(x) = \pi x^2$



- (f) Yes
 45 (a) $P(t) = 150(1.1)^t$
 (b) $P^{-1}(y) = (\log y - \log 150) / \log 1.1$
 (c) 10.3 years

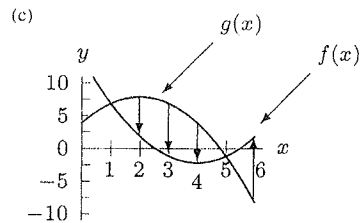
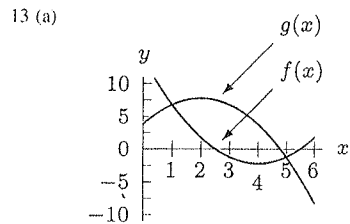
- 47 (a) 36 m
 (b) 6 seconds
 (c) One possibility: $3 \leq t \leq 6$



- 49 (a) A (wrong) guess is 0.99 ml
 (b) $x = C^{-1}(0.98)$
 (c) 50 ml
 51 $f^{-1}(L) = -\frac{1}{k} \ln(1 - L/L_\infty)$
 $f^{-1}(L)$ = Age of fish of length L
 Domain: $0 \leq L \leq L_\infty$

Section 8.3

- 1 $3x^2 + x$
 3 $\sqrt{4x^3 + 8x}$
 5 $3x/2 - 1/2$
 7 (a) $f(x) + g(x) = 3x^2 + x + 1$
 (b) $f(x) - g(x) = -3x^2 + x + 1$
 (c) $f(x)g(x) = 3x^3 + 3x^2$
 (d) $f(x)/g(x) = (x+1)/(3x^2)$
 9 (a) $f(x) + g(x) = 2x$
 (b) $f(x) - g(x) = 10$
 (c) $f(x)g(x) = x^2 - 25$
 (d) $f(x)/g(x) = (x+5)/(x-5)$
 11 (a) $f(x) + g(x) = x^3 + x^2$
 (b) $f(x) - g(x) = x^3 - x^2$
 (c) $f(x)g(x) = x^5$
 (d) $f(x)/g(x) = x$



Section 8.2

- 1 Not invertible
 3 Invertible
 9 $h^{-1}(x) = \sqrt[3]{x/12}$
 11 $k^{-1}(x) = \frac{1}{2} \ln(x/3)$

More Answers
 APPENDIX
 p. 6

PRACTICE # 301
MORE ANSWERS.

More practice Ch. 8.1 - 8.2

8.1 p. 347 = 2, 6, 12, 22*, (42), 46

2.
$$\begin{array}{c|cccc} x & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline f(x) & 2 & 5 & 8 & 3 & 1 & 4 \end{array}$$

6.
$$\frac{1}{(x^2+1)-3} = \frac{1}{x^2-2}$$

12.
$$\frac{1}{x^2-1}$$

22*	x	f(x)	g(x)	f(g(x))
	0	1	2	5
	1	9	0	(1)
	2	(5)	1	(9)

(42). Possibility = $u = x^3, v = 2x+5$.

46. Possibility = $u = \frac{2}{x}, v = 1 + \sqrt{x}$.

8.2 p. 359: 2, 4, 6, 11, (17), 19, 38, 40*

2. Not invertible. 4. Not invertible

6. Check that

$f^{-1}(f(x)) = x$

and $f(f^{-1}(x)) = x$.

(They are.) Or find inverse of f(x).

11. $K^{-1}(x) = \frac{1}{2} (\ln(x/3))$

(17.) $f^{-1}(x) = (4x^2-4)/(x^2-7)$

19. $p^{-1}(x) = 1/e^{(x/2)}$

38. a) $x = -\ln(2)$

b) $x = e^2$

c) $x = 1/\ln(2)$

40* a) $f^{-1}(x) = \frac{2 \sin x}{\sin x + 3}$

b) $g^{-1}(x) = \tan^{-1}(e^x)$

c) $h^{-1}(x) = \cos^{-1}(\sqrt{x}-1)$

b and c have restricted domains.

Extra graphs for 8. Review p. 372 = 35*

a, II. b, I. c, II. d,

e, IV. f,

g, I.

