SHEET #252: SECOND DERIVATIVE & DIFFERENTIABILITY

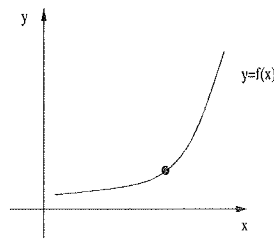
NAME: ______________ Period: ____

GROUP MEMBERS ______________

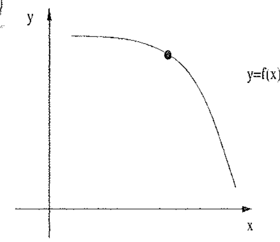
Worksheet 2.5: Concavity and the Second Derivative [1]

1. Answer the following set of questions about each of graphs (a) through (f).
   - i At the indicated point, is f' positive, negative, or zero?
   - ii As you read the graph from left to right, is the *slope* of the graph increasing or decreasing?
   - iii As you read the graph from left to right, is f' increasing or decreasing?
   - iv The derivative of f' is f''. Is f'' positive or negative? Why?
   - v Is the graph *concave up* (curved upward) or *concave down* (curved downward)?

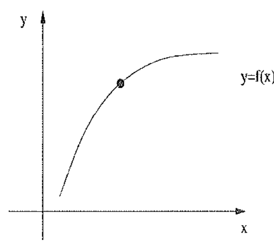
(a)



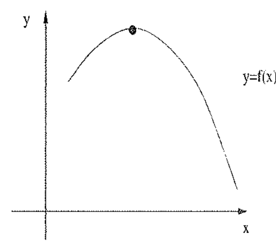
(b)



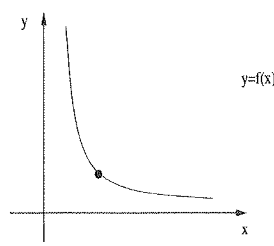
(c)



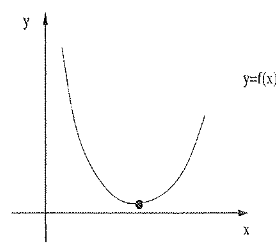
(d)



(e)



(f)



2. Based on your answers to part (iv) and (v), write two rules relating the concavity of the graph of f (up or down) to the sign of the second derivative f'' (positive or negative).

   Rule 1: The graph of f is concave up if and only if f'' is ______________ .

   Rule 2: ____.

[1] Activity by Janet Beery; part 1 adapted from K.R. Hoffman et al., *Calculus in Context* (New York: W.H. Freeman, 1995), pp. 271-272

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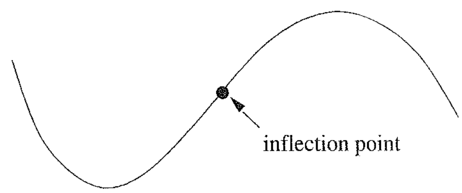
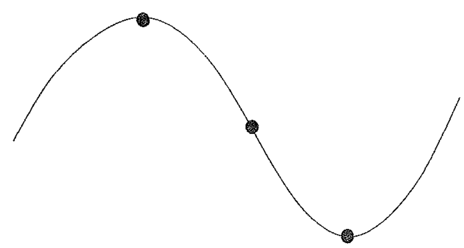
3. A point at which a graph changes concavity is called an inflection point,
Example:



Based on your rules from Exercise 2, if the point $(x, f(x))$ is an inflection point, then $f''(x) =$ ________________ or $f''(x)$ does not exist.

4. The function $f(x) = 2x^3 - 3x^2 - 12x + 8$ is a cubic function (polynomial of degree 3) with leading coefficient 2. Since 2 is positive, the general shape of the graph of $f(x)$ is



Notice that the graph has one local maximum point, one inflection point, and one local minimum point. Find f' and f'', then use them to find these three points. Find both the x-coordinate and y-coordinate of each point. Then find the y-intercept of the graph and sketch the graph. You may use computer graphing software or your calculator to *check* your work.

5. Use $f'(x)$ and $f''(x)$ to find the local minimum point(s), local maximum point(s), and inflection point(s) for the graph of $f(x) = x^4 - 8x^2 + 5$. Sketch the graph, labeling these points with their coordinates.

Worksheet 2.5a: Derivatives

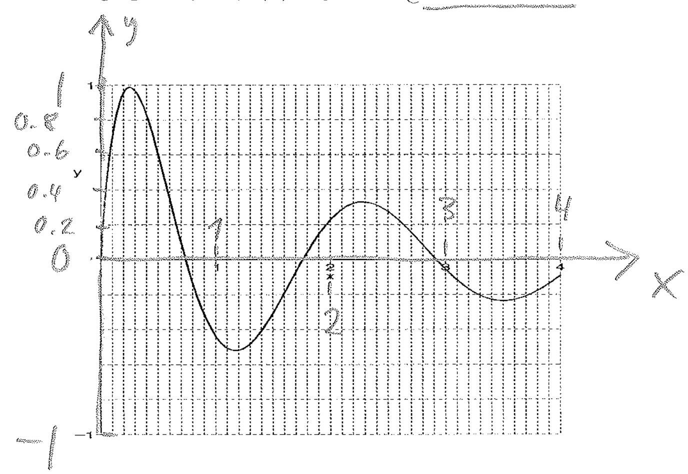
1. In each of the following situations, sketch the graph of a function $f(t)$ that has the indicated properties.

A. $f(t)$ is increasing $f(t) > 0$ $f'(t)$ is increasing	B. $f(t) > 0$ $f(t)$ is decreasing $f''(t) > 0$	C. $f(t)$ is increasing $f''(t) < 0$ $f(t) < 0$	D. $f''(t) < 0$ $f'(t) > 0$ $f(t) < 0$
E. $f(t) < 0$ $f'(t)$ is decreasing $f'(t) < 0$	F. $f'(t)$ is increasing $f(t) > 0$ $f'(t) < 0$	G. $f'(t) < 0$ $f''(t) < 0$ $f(t) > 0$	H. $f(t)$ is increasing $f(t) > 0$ $f''(t) > 0$
I. $f'(t)$ is increasing $f(t)$ is decreasing $f(t) < 0$	J. $f(t) > 0$ $f(t)$ is decreasing $f'(t)$ is decreasing	K. $f(t) < 0$ $f'(t)$ is increasing $f(t)$ is increasing	L. $f''(t) < 0$ $f(t) < 0$ $f(t)$ is decreasing
M. $f(t) > 0$ $f(t)$ is increasing $f''(t) < 0$	N. $f'(t) > 0$ $f(t) > 0$ $f''(t) > 0$	O. $f'(t) < 0$ $f''(t) > 0$ $f(t) > 0$	P. $f(t) < 0$ $f''(t) > 0$ $f'(t) < 0$

Q. $f'(t)$ is decreasing $f(t) < 0$ $f'(t) > 0$	R. $f'(t)$ is increasing $f'(t) > 0$ $f(t) > 0$	S. $f(t) > 0$ $f''(t) < 0$ $f(t)$ is decreasing	T. $f(t)$ is decreasing $f'(t)$ is decreasing $f(t) < 0$
U $f''(t) < 0$ $f(t) > 0$ $f'(t) > 0$	V. $f'(t) > 0$ $f(t) < 0$ $f'(t)$ is increasing	W. $f(t) < 0$ $f'(t) < 0$ $f''(t) < 0$	X. $f'(t)$ is increasing $f'(t) < 0$ $f(t) < 0$
Y. $f(t) < 0$ $f'(t)$ is decreasing $f(t)$ is increasing	Z. $f'(t)$ is decreasing $f(t) > 0$ $f(t)$ is increasing	AA. $f(t)$ is decreasing $f''(t) > 0$ $f(t) < 0$	AB. $f(t)$ is increasing $f(t) < 0$ $f''(t) > 0$
AC. $f''(t) > 0$ $f'(t) > 0$ $f(t) < 0$	AD. $f(t) > 0$ $f'(t)$ is decreasing $f'(t) > 0$	AE. $f'(t) < 0$ $f'(t)$ is decreasing $f(t) > 0$	AF. $f(t) > 0$ $f'(t)$ is increasing $f(t)$ is decreasing

Worksheet 2.5b: The Derivative and Second Derivative

1. Below, the graph of $y = f'(x)$ is given. (Derivative is given!)



y = each minor tick is 0.2
X = each minor tick is 0.1

In all of the questions that follow, estimate to the nearest 0.05 of a unit.

(a) On what interval(s) is f increasing? decreasing? Explain.

INTERVAL(S) MEANS X VALUES SUCH AS $5.2 < x < 7.8$. YOU CAN ALSO MARK THE GRAPH.

(b) On what interval(s) is f' increasing? decreasing? Explain.

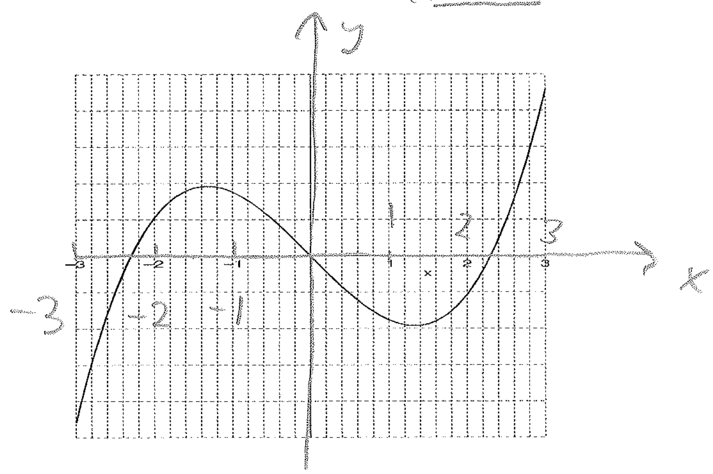
(c) On what interval(s) is f concave up? concave down? Explain.

(d) On what interval(s) is f' concave up? concave down? Explain.

$f(0)=1$ (NOT $f'(0)$)

(e) Given that $f(0) = 1$, what is the approximate equation of the tangent line to $y = f(x)$ at $x = 0.1$? Explain how you arrived at your answer.

2. Below, the graph of $y = g''(x)$ is given. (second derivative is given!)



In all of the questions that follow, estimate to the nearest 0.1 of a unit.

(a) On what interval(s) is $g''(x) > 0$? Explain.

(b) On what interval(s) is $g''(x) < 0$? Explain.

(c) On what interval(s) is g concave up? concave down? Explain.

Worksheet 2.6: Differentiability

1. A magnetic field, B, is given as a function of the distance, r, from the center of a wire as follows:

$$B = \begin{cases} \frac{r}{r_0} B_0 & \text{for } r \leq r_0 \\ \\ \frac{r_0}{r} B_0 & \text{for } r > r_0 \end{cases}$$

   (a) Is B continuous at r_0? Explain.

   (b) Is B differentiable at r_0? Explain.

2. Sketch the graph of $y = f(x)$ if f has the following properties:
   - $f(x)$ is continuous everywhere except at 3
   - $f(x)$ has a vertical tangent line at 2
   - $f(x)$ is not differentiable at 3
   - $f''(x) > 0$ wherever it is defined

3. Find the intersection point of the tangent line to $y = x^x$ at 1.1 and the x-axis.

USE A CALCULATOR. (IT CAN, LATER, BE DONE ALGEBRAICALLY)