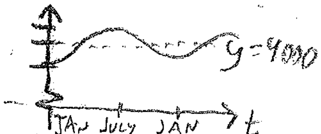
NAME: $\boxed{\text{KEY}}$ PERIOD: ____

Sheet # 237: Rates of change & the derivative

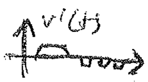
Deborah Hughes-Hallett et al., *Calculus: Single Variable*, Chapter 2.3 page 88.



37. A child inflates a balloon, admires it for a while and then lets the air out at a constant rate. If $V(t)$ gives the volume of the balloon at time t, then Figure 2.34 shows $V'(t)$ as a function of t. At what time does the child:

(a) Begin to inflate the balloon? $\boxed{3s}$
(b) Finish inflating the balloon? $\boxed{9s}$
(c) Begin to let the air out? $\boxed{14s}$
(d) What would the graph of $V'(t)$ look like if the child had alternated between pinching and releasing the open end of the balloon, instead of letting the air out at a constant rate?

(e) When is the balloon the largest? $9 \le t \le 14$ sec.
(f) What is the maximum volume? $\boxed{5dL}$



$V'(t)$ RATE OF CHANGE OF VOLUME VS. TIME (NOT VOLUME ITSELF)

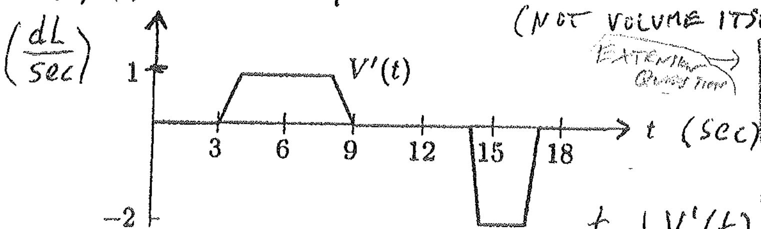


Figure 2.34

Starting volume, $V(0) = 0$ dL

t (sec)	$V'(t)$ (dL/sec)
3	0
4	1
8	1
9	0
14	0
14.5	−2
16.5	−2
17	0

RADIUS ≈ 5 cm.
$V \approx 500\ cm^3$
1 dL =
1 deciLiter = $\frac{1}{10}$ Liter ≈ $\frac{1}{5}$ pint.

EXTENSION QUESTION: 37g, Let $R(t)$ = RADIUS OF A BALLOON. IF $V'(t)$ = CONSTANT, SKETCH $R(t)$ and $R'(t)$.

$V = \frac{4}{3}\pi R^3$
$V' = 4\pi R^2 R' = $ const.
$R' = $ const$/4\pi R^2$

38. The population of a herd of deer is modeled by (MONTHS)

$$P(t) = 4000 + 500 \sin\left(2\pi t - \frac{\pi}{2}\right)$$

where t is measured in years from January 1.

(a) How does this population vary with time? Sketch a graph of $P(t)$ for one year. $\boxed{\text{SINUSOIDALLY WITH PERIOD = 1 YEAR.}}$
(b) Use the graph to decide when in the year the population is a maximum. What is that maximum? Is there a minimum? If so, when? $\boxed{\text{JULY 1, 4500}}$ $\boxed{\text{JAN 1, 3500}}$
(c) Use the graph to decide when the population is growing fastest. When is it decreasing fastest? $\boxed{\text{APRIL 1, OCT 1.}}$
(d) Estimate roughly how fast the population is changing on the first of July. $\boxed{\text{Slope = 0.}}$

39. The graph in Figure 2.35 shows the accumulated federal debt since 1970. Sketch the derivative of this function. What does it represent?

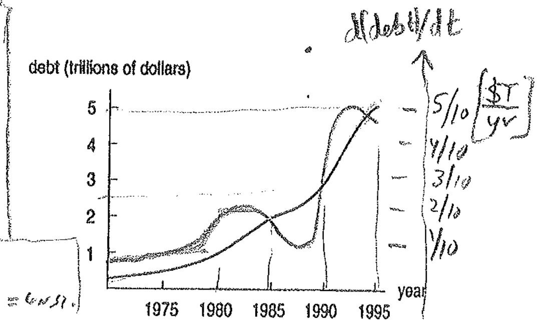
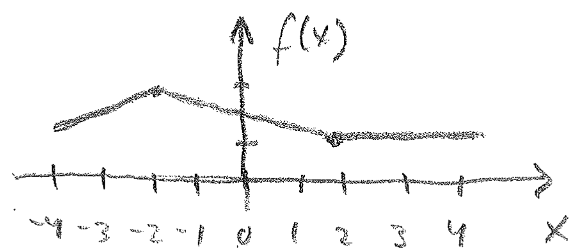


Figure 2.35

1/10 1/5 1/10 1/2
0.1 0.2 0.1 0.5

40. Draw the graph of a continuous function $y = f(x)$ that satisfies the following three conditions.

- $f'(x) > 0$ for $x < -2$,
- $f'(x) < 0$ for $-2 < x < 2$,
- $f'(x) = 0$ for $x > 2$.



Barron's *How to Prepare for the AP Calculus*. Chapter 3, SET 3, p. 74. 8th Edition.

66. A differentiable function f has the values shown. Estimate $f'(1.5)$.

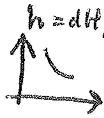
x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

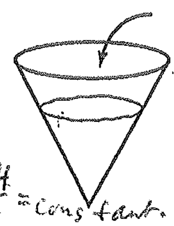
$\frac{22-14}{1.6-1.4} = \frac{8}{0.2} = 40$

(A) 8 (B) 12 (C) 18 **(D) 40** (E) 80

67. Water is poured into a conical reservoir at a constant rate. If $h(t)$ is the rate of change of the depth of the water, then h is

$h = dH/dt$





(A) constant
(B) linear and increasing
(C) linear and decreasing
(D) nonlinear and increasing
(E) nonlinear and decreasing

$V = kH^3$

$\frac{dV}{dt} = 3kH^2 \cdot \frac{dH}{dt} = $ constant.

RATE $= h(t) = dH/dt$ is proportional to $1/H^2$

Vocabulary: Derivative = instantaneous rate of change = slope $= f'(x) = \frac{\Delta f(x)}{\Delta x}$ as $\Delta x \to 0$ $= \frac{df(x)}{dx}$.

37g.

$$\frac{dR}{dt} = \frac{K}{R^2}$$

$$\int R^2\,dR = \int K\,t$$

$$\frac{R^3}{3} = Kt + c$$

$$R = \sqrt[3]{3(Kt+c)}$$

$$R(0) = 0$$

$$0 = \sqrt[3]{3(K\cdot 0 + c)}$$

$$0 = 3(k\cdot 0 + c)$$

$$0 = 0 + c$$

$$R(t) = \sqrt[3]{3Kt} = (3Kt)^{1/3}$$

$$R' = \frac{1}{3}(3Kt)^{-2/3}\cdot 3K$$

$$= \frac{K}{(3Kt)^{2/3}} = \frac{1}{\sqrt[3]{t^2}}$$