NAME: ____________________ PERIOD: ____

Sheet # 237: Rates of change & the derivative

37(g) Let R(t) = RADIUS OF THE BALLOON, IF V'(t) = CONSTANT, SKETCH R(t) AND R'(t).

37. A child inflates a balloon, admires it for a while and then lets the air out at a constant rate. If $V(t)$ gives the volume of the balloon at time t, then Figure 2.34 shows $V'(t)$ as a function of t. At what time does the child:

(a) Begin to inflate the balloon?
(b) Finish inflating the balloon?
(c) Begin to let the air out?
(d) What would the graph of $V'(t)$ look like if the child had alternated between pinching and releasing the open end of the balloon, instead of letting the air out at a constant rate?

(e) When is the balloon the largest?
(f) What is the maximum volume?
(g)

$V'(t)$ RATE OF CHANGE OF VOLUME VS. TIME (NOT VOLUME ITSELF)

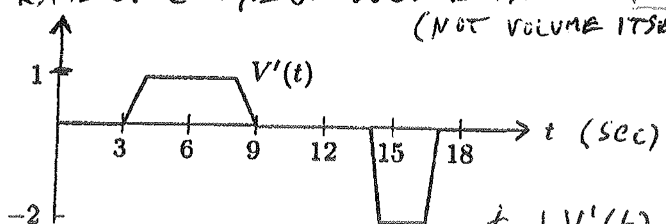


Figure 2.34

Starting volume, $V(0) = 0$ dL

t (sec)	$V'(t)$ (dL/sec)
3	0
4	1
8	1
9	0
14	0
14.5	−2
16.5	−2
17	0

1 dL = 1 deciLiter = $\frac{1}{10}$ Liter ≈ $\frac{1}{5}$ pint.

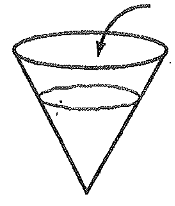
66. A differentiable function f has the values shown. Estimate $f'(1.5)$.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

(A) 8 (B) 12 (C) 18 (D) 40 (E) 80

67. Water is poured into a conical reservoir at a constant rate. If $h(t)$ is the rate of change of the depth of the water, then h is

(A) constant
(B) linear and increasing
(C) linear and decreasing
(D) nonlinear and increasing
(E) nonlinear and decreasing



SKETCH $h(t)$

38. The population of a herd of deer is modeled by

$$P(t) = 4000 + 500 \sin\left(2\pi t - \frac{\pi}{2}\right)$$

where t is measured in years from January 1.

(a) How does this population vary with time? Sketch a graph of $P(t)$ for one year.
(b) Use the graph to decide when in the year the population is a maximum. What is that maximum? Is there a minimum? If so, when?
(c) Use the graph to decide when the population is growing fastest. When is it decreasing fastest?
(d) Estimate roughly how fast the population is changing on the first of July.

39. The graph in Figure 2.35 shows the accumulated federal debt since 1970. Sketch the derivative of this function. What does it represent?

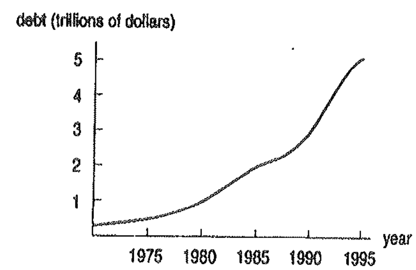


Figure 2.35

40. Draw the graph of a continuous function $y = f(x)$ that satisfies the following three conditions.

- $f'(x) > 0$ for $x < -2$,
- $f'(x) < 0$ for $-2 < x < 2$,
- $f'(x) = 0$ for $x > 2$.

Vocabulary: Derivative = instantaneous rate of change = slope = $f'(x)$