SHEET #254 : MOTION EXPLORATIONS. NAME = Period: _

Refer to Sheet #253 (F3.4)

Exploration 2 Modeling Horizontal Motion

The position (*x*-coordinate) of a particle moving on the horizontal line $y = 2$ is given by $x(t) = 4t^3 - 16t^2 + 15t$ for $t \geq 0$.

V = Velocity
a = acceler.
S = x(t) = POSITION

1. Graph the parametric equations $x_1(t) = 4t^3 - 16t^2 + 15t$, $y_1(t) = 2$ in $[-4, 6]$ by $[-3, 5]$. Use TRACE to support that the particle starts at the point (0, 2), moves to the right, then to the left, and finally to the right. At what times does the particle reverse direction?

t VS. S = x(t)

2. Graph the parametric equations $x_2(t) = x_1(t)$, $y_2(t) = t$ in the same viewing window. Explain how this graph shows the back and forth motion of the particle. Use this graph to find when the particle reverses direction.

USUAL GRAPH: S = x(t) VS. t

3. Graph the parametric equations $x_3(t) = t$, $y_3(t) = x_1(t)$ in the same viewing window. Explain how this graph shows the back and forth motion of the particle. Use this graph to find when the particle reverses direction.

FIND CALC dy/dt for t = 1

4. Use the methods in parts 1, 2, and 3 to represent and describe the *velocity* of the particle.

EXPLORATION 2

MODE RADIAN, PAR, DOT

Y=

$X_{1T} = 4T^3 - 16T^2 + 15T$

$Y_{1T} = 2$

WINDOW

TMIN = 0
TMAX = 3
TSTEP = 0.05
XMIN = −4 YMIN = −3
XMAX = 6 YMAX = 5
XSCL = 1 YSCL = 1

5. MODE FUNC, CONNECTED

Y= $Y_1 = 4X^3 - 16X^2 + 1$

WINDOW YMIN = −7
XRES = 4

5. GRAPH FUNCTIONS: $Y_1 = S$, $Y_2 = V$, $Y_3 = a$.

- FIND MAX & MIN POSITIONS WITH TIMES,
- FIND MINIMUM VELOCITY AND THE TIME.
- FIND VELOCITY AT t = 1.

Exploration 3 Seeing Motion on a Graphing Calculator

The graphs in Figure 3.26b give us plenty of information about the flight of the rock in Example 4, but neither graph shows the path of the rock in flight. We can simulate the moving rock by graphing the parametric equations

$$x_1(t) = 3(t < 5) + 3.1(t \geq 5), \qquad y_1(t) = 160t - 16t^2$$

in dot mode.

USE 3 + 0.1T INSTEAD.
HORIZONTAL VELOCITY IS 0.1 ft/sec.

This will show the upward flight of the rock along the vertical line $x = 3$, and the downward flight of the rock along the line $x = 3.1$.

UP FROM X=3 TO DOWN AT X=4.

1. To see the flight of the rock from beginning to end, what should we use for *t*Min and *t*Max in our graphing window?
2. Set *x*Min = 0, *x*Max = 6, and *y*Min = −10. Use the results from Example 4 to determine an appropriate value for *y*Max. (You will want the entire flight of the rock to fit within the vertical range of the screen.)

→ 0.2 INSTEAD, THEN TRY 0.1 AND 0.3

3. Set *t*Step initially at 0.1. (A higher number will make the simulation move faster. A lower number will slow it down.)
4. Can you explain why the grapher actually slows down when the rock would slow down, and speeds up when the rock would speed up?

IF SO, EXPLAIN!

EXPLORATION 3

MODE RADIAN, PAR, DOT

Y= $X_{5T} = 3 + 0.1T$
$Y_{5T} = 160T - 16T^2$

QUESTION 2

WINDOW ← QUESTION 1.

TMIN = [ ]	XMIN = 0	YMIN = −10 ↓
TMAX = [ ]	XMAX = 6	YMAX = [ ]
TSTEP = 0.2	XSCL = 1	YSCL = 100

EXPLORATION 2 ANSWERS TO QUESTION 5.

$V(t=1) = 12 - 32 + 15 = -5$

$V = 12t^2 - 32t + 15$

$S_{MIN} = (2.060, -2.030)$

$S_{MAX} = (0.607, 4.107)$

$a = 24t - 32$

$a = 0 \rightarrow t = 32/24 = 4/3 \approx 1.333$

$V_{MIN} = -6\frac{1}{3} \approx -6.333$

WHEN $t = 1.333$, $S = 28/27 \approx 1.037$