NAME: _____ PERIOD: ____

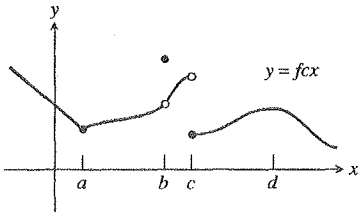
Sheet #182: Chapter 1.6 to 1.8 Review: Rational Functions, Limits and Continuity.
Required: 2, 5, 6, 7, 8, 15–20, 21–24, 25, 27, 28, 29, 31, 37, 39, 40, 41, 42, 52.
Recommended: 11, 12, 13, 14, 26, 30, 33–35.
Hand in packet at the time of the test. Show work when appropriate and use blank spaces, or extra pages.

In Exercises 1–14, find the limits.

1. $\lim_{x\to -2} (x^3 - 2x^2 + 1)$
2. $\lim_{x\to -2} \dfrac{x^2+1}{3x^2-2x+5}$
3. $\lim_{x\to 4} \sqrt{1-2x}$
4. $\lim_{x\to 5} \sqrt[4]{9-x^2}$
5. $\lim_{x\to 0} \dfrac{\frac{1}{2+x} - \frac{1}{2}}{x}$
6. $\lim_{x\to \pm\infty} \dfrac{2x^2+3}{5x^2+7}$
7. $\lim_{x\to \pm\infty} \dfrac{x^4+x^3}{12x^3+128}$
8. $\lim_{x\to 0} \dfrac{\sin 2x}{4x}$
9. $\lim_{x\to 0} \dfrac{x\csc x + 1}{x\csc x}$
10. $\lim_{x\to 0} e^x \sin x$
11. $\lim_{x\to 7/2^+} \text{int}\,(2x-1)$
12. $\lim_{x\to 7/2^-} \text{int}\,(2x-1)$
13. $\lim_{x\to \infty} e^{-x}\cos x$
14. $\lim_{x\to \infty} \dfrac{x+\sin x}{x+\cos x}$

In Exercises 15–20, determine whether the limit exists on the basis of the graph of $y = f(x)$. The domain of f is the set of real numbers.

15. $\lim_{x\to d} f(x)$
16. $\lim_{x\to c^+} f(x)$
17. $\lim_{x\to c^-} f(x)$
18. $\lim_{x\to c} f(x)$
19. $\lim_{x\to b} f(x)$
20. $\lim_{x\to a} f(x)$



In Exercises 21–24, determine whether the function f used in Exercises 15–20 is continuous at the indicated point.

21. $x = a$
22. $x = b$
23. $x = c$
24. $x = d$

In Exercises 25 and 26, use the graph of the function with domain $-1 \le x \le 3$.

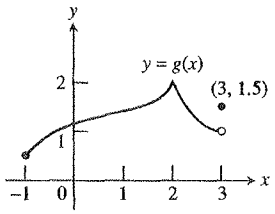
25. Determine

(a) $\lim_{x\to 3^-} g(x)$. (b) $g(3)$.

(c) whether $g(x)$ is continuous at $x = 3$.

(d) the points of discontinuity of $g(x)$.

(e) Writing to Learn whether any points of discontinuity are removable. If so, describe the extended function. If not, explain why not.



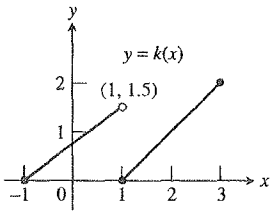
26. Determine

(a) $\lim_{x\to 1^-} k(x)$. (b) $\lim_{x\to 1^+} k(x)$. (c) $k(1)$.

(d) whether $k(x)$ is continuous at $x = 1$.

(e) the points of discontinuity of $k(x)$.

(f) Writing to Learn whether any points of discontinuity are removable. If so, describe the extended function. If not, explain why not.



In Exercises 27 and 28, (a) find the vertical asymptotes of the graph of $y = f(x)$, and (b) describe the behavior of $f(x)$ to the left and right of any vertical asymptote.

27. $f(x) = \dfrac{x+3}{x+2}$
28. $f(x) = \dfrac{x-1}{x^2(x+2)}$

In Exercises 29 and 30, answer the questions for the piecewise-defined function.

29. $f(x) = \begin{cases} 1, & x \le -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$

(a) Find the right-hand and left-hand limits of f at $x = -1$, 0, and 1.

(b) Does f have a limit as x approaches -1? 0? 1? If so, what is it? If not, why not?

(c) Is f continuous at $x = -1$? 0? 1? Explain.

30. $f(x) = \begin{cases} |x^3 - 4x|, & x < 1 \\ x^2 - 2x - 2, & x \ge 1 \end{cases}$

(a) Find the right-hand and left-hand limits of f at $x = 1$.

(b) Does f have a limit as $x \to 1$? If so, what is it? If not, why not?

(c) At what points is f continuous?

(d) At what points is f discontinuous?

In Exercises 31 and 32, find all points of discontinuity of the function.

31. $f(x) = \dfrac{x + 1}{4 - x^2}$ **32.** $g(x) = \sqrt[3]{3x + 2}$

In Exercises 33–36, find **(a)** a power function end behavior model and **(b)** any horizontal asymptotes.

33. $f(x) = \dfrac{2x + 1}{x^2 - 2x + 1}$ **34.** $f(x) = \dfrac{2x^2 + 5x - 1}{x^2 + 2x}$

35. $f(x) = \dfrac{x^3 - 4x^2 + 3x + 3}{x - 3}$ **36.** $f(x) = \dfrac{x^4 - 3x^2 + x - 1}{x^3 - x + 1}$

In Exercises 37 and 38, find **(a)** a right end behavior model and **(b)** a left end behavior model for the function.

37. $f(x) = x + e^x$ **38.** $f(x) = \ln|x| + \sin x$

In Exercises 39 and 40, *work in groups of two or three.* What value should be assigned to k to make f a continuous function?

39. $f(x) = \begin{cases} \dfrac{x^2 + 2x - 15}{x - 3}, & x \ne 3 \\ k, & x = 3 \end{cases}$

40. $f(x) = \begin{cases} \dfrac{\sin x}{2x}, & x \ne 0 \\ k, & x = 0 \end{cases}$

In Exercises 41 and 42, *work in groups of two or three.* Sketch a graph of a function f that satisfies the given conditions.

41. $\lim_{x \to \infty} f(x) = 3$, $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to 3^+} f(x) = \infty$, $\lim_{x \to 3^-} f(x) = -\infty$

42. $\lim_{x \to 2} f(x)$ does not exist, $\lim_{x \to 2^+} f(x) = f(2) = 3$

43. *Average Rate of Change* Find the average rate of change of $f(x) = 1 + \sin x$ over the interval $[0, \pi/2]$.

44. *Rate of Change* Find the instantaneous rate of change of the volume $V = (1/3)\pi r^2 H$ of a cone with respect to the radius r at $r = a$ if the height H does not change.

45. *Rate of Change* Find the instantaneous rate of change of the surface area $S = 6x^2$ of a cube with respect to the edge length x at $x = a$.

46. *Slope of a Curve* Find the slope of the curve $y = x^2 - x - 2$ at $x = a$.

47. *Tangent and Normal* Let $f(x) = x^2 - 3x$ and $P = (1, f(1))$. Find **(a)** the slope of the curve $y = f(x)$ at P, **(b)** an equation of the tangent at P, and **(c)** an equation of the normal at P.

48. *Horizontal Tangents* At what points, if any, are the tangents to the graph of $f(x) = x^2 - 3x$ horizontal? (See Exercise 47)

49. *Bear Population* The number of bears in a federal wildlife reserve is given by the population equation

$$p(t) = \frac{200}{1 + 7e^{-0.1t}},$$

where t is in years.

(a) Writing to Learn Find $p(0)$. Give a possible interpretation of this number.

(b) Find $\lim_{t \to \infty} p(t)$.

(c) Writing to Learn Give a possible interpretation of the result in (b).

50. *Taxi Fares* Bluetop Cab charges \$3.20 for the first mile and \$1.35 for each additional mile or part of a mile.

(a) Write a formula that gives the charge for x miles with $0 \le x \le 20$.

(b) Graph the function in (a). At what values of x is it discontinuous?

51. *Congressional Academic Funding* Consider Table 2.4 in Exercise 34 of Section 2.4.

(a) Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth. Find a cubic and a quartic regression equation for the data.

(b) Find an end behavior model for each regression equation. What does each predict about future funding?

52. *Limit Properties* Assume that

$$\lim_{x \to c} [f(x) + g(x)] = 2,$$

$$\lim_{x \to c} [f(x) - g(x)] = 1,$$

and that $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist. Find $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$.

53. Table 2.5 gives the spending for software, equipment, and services to create and run *intranets*—private, Internet-like networks that link a company's operations.

(a) Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. Make a scatter plot of the data.

(b) Let P represent the point corresponding to 2000, and Q the point for any one of the previous years. Make a table of the slopes possible for the secant line PQ.

(c) Predict the rate of change of spending in 2000.

(d) Find a quadratic regression equation for the data, and use it to calculate the rate of change of spending in 2000.

Table 2.5 Intranet Spending

Year	Spending (\$ billions)
1995	2.7
1996	4.8
1997	7.8
1998	11.2
1999	15.2
2000	20.1

Source: Killen & Associates as reported by Anne R. Carey and Elys A. McLean in *USA Today*, April 14, 1997.

Sheet #18

CHAPTER 1.6 – 1.8 REVIEW ANSWERS.

RATIONAL FUNCTIONS

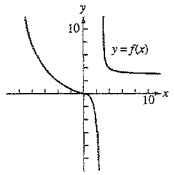
LIMITS

CONTINUITY.

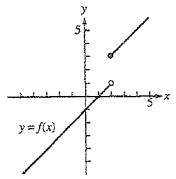
REVIEW

Chapter Review (pp. 91–93)

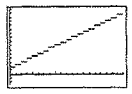
1. -15
2. $\frac{5}{21}$
3. No limit
4. No limit
5. $-\frac{1}{4}$
6. $\frac{2}{5}$
7. $+\infty, -\infty$
8. $\frac{1}{2}$
9. 2
10. 0
11. 6
12. 5
13. 0
14. 1
15. Limit exists
16. Limit exists
17. Limit exists
18. Doesn't exist
19. Limit exists
20. Limit exists
21. Yes
22. No
23. No
24. Yes
25. (a) 1 (b) 1.5 (c) No
    (d) g is discontinuous at $x = 3$ (and points not in domain).
    (e) Yes, can remove discontinuity at $x = 3$ by assigning the value 1 to $g(3)$.
26. (a) 1.5 (b) 0
    (c) 0 (d) No
    (e) k is discontinuous at $x = 1$ (and points not in domain).
    (f) Discontinuity at $x = 1$ is not removable because the two one-sided limits are different.
27. (a) Vertical Asymp.: $x = -2$
    (b) Left-hand limit $= -\infty$
    Right-hand limit $= \infty$
28. (a) Vertical Asymp.: $x = 0$ and $x = -2$
    (b) At $x = 0$:
    Left-hand limit $= -\infty$
    Right-hand limit $= -\infty$
    At $x = -2$:
    Left-hand limit $= \infty$
    Right-hand limit $= -\infty$
29. (a) At $x = -1$:
    Left-hand limit $= 1$
    Right-hand limit $= 1$
    At $x = 0$:
    Left-hand limit $= 0$
    Right-hand limit $= 0$
    At $x = 1$:
    Left-hand limit $= -1$
    Right-hand limit $= 1$
    (b) At $x = -1$:
    Yes, the limit is 1.
    At $x = 0$:
    Yes, the limit is 0.
    At $x = 1$:
    No, the limit doesn't exist because the two one-sided limits are different.
    (c) At $x = -1$:
    Continuous because $f(-1) =$ the limit.
    At $x = 0$:
    Discontinuous because $f(0) \neq$ the limit.
    At $x = 1$:
    Discontinuous because limit doesn't exist.
30. (a) Left-hand limit $= 3$
    Right-hand limit $= -3$
    (b) No, because the two one-sided limits are different.
    (c) Every place except for $x = 1$
    (d) At $x = 1$
31. $x = -2$ and $x = 2$
32. There are no points of discontinuity.
33. (a) $\frac{2}{x}$ (b) $y = 0$ (x-axis)
34. (a) 2 (b) $y = 2$
35. (a) x^2 (b) None
36. (a) x (b) None
37. (a) e^x (b) x
38. (a) $\ln|x|$ (b) $\ln|x|$
39. $k = 8$
40. $k = \frac{1}{2}$
41. One possible answer:



42. One possible answer:



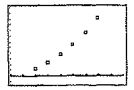
43. $\frac{2}{\pi}$
44. $\frac{2}{3}\pi aH$
45. $12a$
46. $2a - 1$
47. (a) -1 (b) $y = -x - 1$
    (c) $y = x - 3$
48. $\left(\frac{3}{2}, -\frac{9}{4}\right)$
49. (a) 25. Perhaps this is the number of bears placed in the reserve when it was established.
    (b) 200
    (c) Perhaps this is the maximum number of bears which the reserve can support due to limitations of food, space, or other resources. Or, perhaps the number is capped at 200 and excess bears are moved to other locations.
50. (a)
$$f(x) = \begin{cases} 3.20 - 1.35 \cdot \text{int}(-x + 1), & 0 < x \leq 20 \\ 0, & x = 0 \end{cases}$$
    (b)



[0, 20] by [−5, 32]

f is discontinuous at integer values of x: 0, 1, 2, ..., 19

51. (a) Cubic:
$y = -1.644x^3 + 42.981x^2 - 254.369x + 300.232$
Quartic:
$y = 2.009x^4 - 102.081x^3 + 1884.997x^2 - 14918.180x + 43004.464$
    (b) Cubic: $-1.644x^3$, predicts spending will go to 0.
Quartic: $2.009x^4$, predicts spending will go to ∞.
52. $\lim_{x \to c} f(x) = \frac{3}{2}$, $\lim_{x \to c} g(x) = \frac{1}{2}$
53. (a)



[3, 12] by [−2, 24]

    (b)

Year of Q	Slope of PQ
1995	3.48
1996	3.825
1997	4.1
1998	4.45
1999	4.9

    (c) Approximately 5 billion dollars per year
    (d) $y = 0.3214x^2 - 1.3471x + 1.3857$
Predicted rate of change in 2000 is 5.081 billion dollars per year.