SHEET #162 :

RATIONAL FUNCTIONS :

3 CASES OF END BEHAVIOR (= LONG RUN BEHAVIOR) including HORIZONTAL AND OTHER ASYMPTOTES.

Let $p(x) = a_n x^n + \cdots + a_1 x + a_0$

and $q(x) = b_m x^m + \cdots + b_1 x + b_0$

$f(x) = \dfrac{p(x)}{q(x)}$ = RATIO OF POLYNOMIALS = **RATIONAL FUNCTION**

$a_n x^n$ is leading term of $p(x)$

$b_m x^m$ is leading term of $q(x)$

n is degree of $p(x)$. a_n is leading coefficient of $p(x)$.

m is degree of $q(x)$ b_m is leading coefficient of $q(x)$.

GENERAL METHOD DIVIDE LEADING TERMS = $\dfrac{a_n x^n}{b_m x^m}$

AND TAKE LIMIT AS $x \to \pm\infty$

TASK: FIND END BEHAVIOR AND/OR HORIZONTAL Asymptote for $y = f(x) = p(x)/q(x)$.

CASE 1. $n = m$ Degree of $p(x)$ = Degree of $q(x)$

EX. $f(x) = \dfrac{3x^3 + 2x + 7}{4x^3 + 8}$

END BEHAVIOR FUNCTION: $y = \dfrac{3x^3}{4x^3} = \dfrac{3}{4}$. [LIMIT IS A NUMBER = 3/4.]

HORIZONTAL Asymptote $\boxed{y = \frac{3}{4}}$ [THE EQUATION OF A FUNCTION.]

CASE 2. $n > m$ Degree of $p(x)$ = Degree of $q(x)$

EX. $f(x) = \dfrac{3x^4 + 2x + 7}{4x^3 + 8}$

END BEHAVIOR FUNCTION: $y = \dfrac{3x^4}{4x^3}$ $\boxed{y = \frac{3}{4}x}$

SINCE $y = \frac{3}{4}x$ IS A LINE IT IS CALLED A "SLANT" ASYMPTOTE*

THE LIMIT IS $y \to \boxed{\pm\infty}$ (NO HORIZONTAL ASYMPTOTE)

CASE 3. $n < m$ Degree of $p(x)$ < Degree of $q(x)$

EX. $f(x) = \dfrac{3x^3 + 2x + 7}{4x^4 + 8}$. END BEHAVIOR FUNCTION: $y = \dfrac{3x^3}{4x^4}$. $\boxed{y = \frac{3}{4}\frac{1}{x}}$

HORIZONTAL ASYMPTOTE $\boxed{y = 0}$ [because $\frac{\text{BIG \#}}{\text{SMALL \#}}$ = SMALL #]

* IF END BEHAVIOR FUNCTION IS NOT A LINE, IT IS USUALLY NOT CALLED AN ASYMPTOTE. (ALTHOUGH IT COULD BE CALLED "NONLINEAR" ASYMPTOTE.) Ex. $f(x) = \dfrac{3x^5 + 2x + 7}{4x^3 + 8}$ has...

... this END BEHAVIOR FUNCTION: $y = \dfrac{3x^5}{4x^3}$. $\boxed{y = \frac{3}{4}x^2}$

NOTE THAT ALL END BEHAVIOR FUNCTIONS ARE PROPORTIONAL TO THE RATIO OF LEADING COEFFICIENTS (except $y=0$) $y = \frac{3}{4}\frac{1}{x}$, $y = \frac{3}{4}$, $y = \frac{3}{4}x$, $y = \frac{3}{4}x^2$.