

FULL NAME: _____

KEY

Period: _____

9/6/07, v.1.

"Sheet #220"

Group Members: _____

FINDING A ZERO WITHOUT A CALCULATOR | A CUBIC POLYNOMIAL

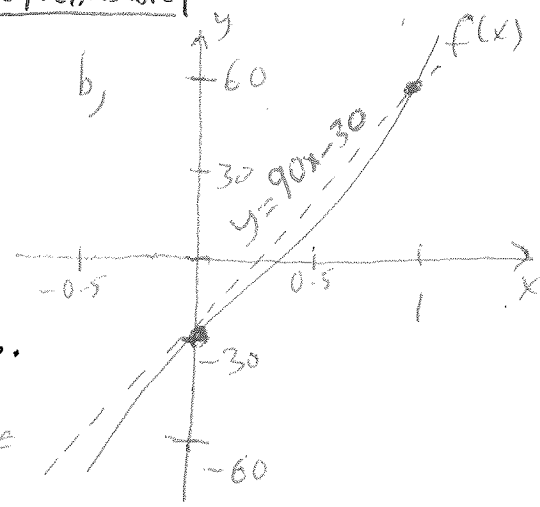
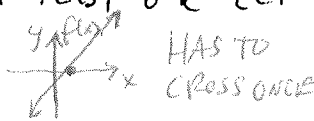
You are charged with finding a zero of the function

$$f(x) = 54x^3 - 40x^2 + 76x - 30.$$

You may not use a calculator to solve this problem. Put the TI-83 away!

a, Explain why $f(x)$ has at least one zero.

$$f(x) \rightarrow \infty \text{ as } x \rightarrow \infty$$
$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -\infty$$



b, Complete the table, Make a sketch!

x	f(x)
1	60
0	-30
-1	-200

$$54 - 40 + 76 - 30 = 14 + 46 = 60$$

$$54(-1)^3 - 40(-1)^2 + 76(-1) - 30 = -54 - 40 - 76 - 30 = -200$$

c, Between what x-values is a good place to look for zeros? $0 < x < 1$

Explain a "good enough" method, that does not involve a lot of computation, for estimating the zero to 1 decimal. Give your estimate.

Hint: lines or proportional reasoning may be useful.

$$\text{LINE} = y - (-30) = \left(\frac{60 - (-30)}{1 - 0}\right)(x - 0) \rightarrow y = 90x - 30 = 0$$

PT. SLOPE FORMULA FOR $(0, -30)$.

$$x = 1/3 \approx 0.333$$

FIRST ESTIMATE

$$x \approx \underline{0.3}$$

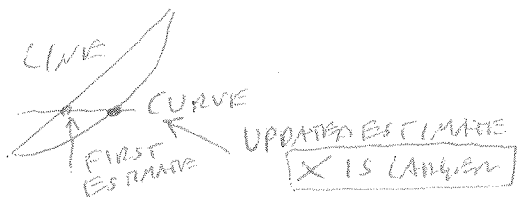
(1 decimal)

d, A reliable source tells you that

the function $f(x)$ is concave up from $x=0.25$ to $x=1.00$.

How could this knowledge be used to improve your estimate? Explain.

What is your "final" answer to the question c?



UPDATED ESTIMATE

$$x \approx \underline{0.4}$$

e, You get hold of a non-graphing CALCULATOR. How would you find the zero to 2 decimals?

f, If you get this far, ask for permission

to use a TI-83 calculator to find the zero to 3 decimals. $x \approx 0.436$

PERMISSION (TEACHERS INITIALS)	
--------------------------------------	--

GRAPH $[-1, 1] \times [-120, 60]$
USE 2ND/CALC 2: ZERO.

g, then compute $x^2 \approx 0.190$

MAKE A CONJECTURE FOR THE EXACT ZERO. $x = \sqrt{0.190} \approx 0.43589$

CHECK YOUR ANSWER USING $f(x)$. $f(\sqrt{0.190}) = -1.4 \cdot 10^{-4} = -0.00014$

TRIAL & ERROR ABOUT 0.3 to 0.4.