SCAN

NAME: KEY Period: ____

SHEET #1151:

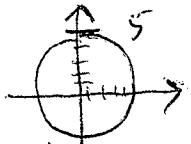
SHAPES, METHODS, CONNECTIONS

CONIC SECTIONS, SEPARATION OF VARIABLES, IMPLICIT DIFFERENTIATION EQUATIONS ARE RELATIONS!

1. GIVEN EQUATIONS, GIVE NAMES OF SHAPES, MAKE A SIMPLE SKETCH & STATE "FEATURES"

a) $x^2+y^2=25$

NAME: CIRCLE



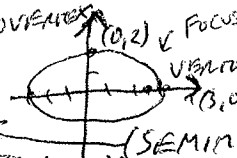
WHAT DOES "25" MEAN? $r^2=(\text{RADIUS})^2=25$

FEATURE(S)?
RADIUS $=r=5$

b) $x^2/9+y^2/4=1$

NAME: ELLIPSE

"9" = a^2 "4" = b^2



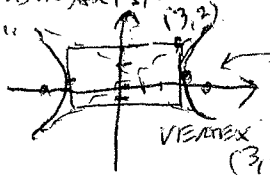
$c=\sqrt{a^2-b^2}=\sqrt{5}\approx 2.24$

FEATURES?
HORIZONTAL
$a=3,\ b=2$
(SEMIMINOR AXIS)^2
(SEMIMAJOR AXIS)^2

c) $x^2/9-y^2/4=1$

NAME: HYPERBOLA

"9"? "4"?



ASYMPTOTES: $y=\pm\frac{2}{3}x$.

FEATURES?
HORIZONTAL
FOCUS, $c=\sqrt{a^2+b^2}=\sqrt{9+4}=\sqrt{13}=3.60$

2. GIVEN DERIVATIVE, FIND A RELATION (NOT NECESSARILY A FUNCTION) BETWEEN x AND y.

$dy/dx=-\frac{x}{y}$, $y(0)=5$

a) WHAT METHOD WOULD YOU USE? (BE SPECIFIC IN THE NAME OF THE DIFFERENTIAL EQUATION WITH INITIAL CONDITION SOLVED BY MAIN METHOD)

SEPARATION OF VARIABLES

b) FIND THE RELATION (BETWEEN x and y)

c) WHAT IS SHAPE? CIRCLE

① Separate
$y\,dy=-x\,dx$

② INTEGRALS
$\int y\,dy=-\int x\,dx$

③ ANALYTIC INTEGRATION
$\frac{y^2}{2}=-\frac{x^2}{2}+C_1$

④ Redefine C
$y^2=-x^2+C$

⑤ Reorganize
$x^2+y^2=C$

⑥ INITIAL CONDITION
$0^2+5^2=C$

⑦ ANSWER
$x^2+y^2=25$
See part 1a.

3. GIVEN RELATION, FIND DERIVATIVE.

$x^2/9+y^2/4=1$

a) WHAT METHOD WOULD YOU USE? (BE SPECIFIC IN THE NAME OF THE MAIN METHOD)

IMPLICIT DIFFERENTIATION

b) FIND THE DERIVATIVE dy/dx. (NOT NECESSARILY A FUNCTION.)

① Differentiate both sides
$\frac{d}{dx}(x^2/9)+\frac{d}{dx}(y^2/4)=\frac{d}{dx}(1)$

② CHAIN RULE
$2x/9+(2y/4)\frac{dy}{dx}=0$

③ Collect terms, factor dy/dx
$(2y/4)\frac{dy}{dx}=-2x/9$

④ Solve for dy/dx
$\frac{dy}{dx}=\frac{-2x/9}{2y/4}$

⑤ Reorganize
$\frac{dy}{dx}=-\frac{4}{9}\left(\frac{x}{y}\right)$.