

Name: \_\_\_\_\_

KEY

Period: \_\_\_\_\_

## Sheet #556: Exponential Equations and Exponential Models

Possibly helpful formulas:  $y = A\left(1 + \frac{r}{n}\right)^{nt}$      $y = Ae^{kt}$      $y = A\left(\frac{1}{2}\right)^{t/T}$

1. Solve for
- $x$
- .

$2^{x+1} = 5$

$$\begin{aligned} \ln(2^{x+1}) &= \ln(5) \\ (x+1)\ln(2) &= \ln(5) \\ x+1 &= \ln(5)/\ln(2) \end{aligned}$$

$$\begin{aligned} x &= \frac{\ln(5)}{\ln(2)} - 1 \\ &\approx 1.322 \end{aligned}$$

2. Solve for
- $t$
- .

$e^{0.1t} = 4$

$$\begin{aligned} \ln(e^{0.1t}) &= \ln(4) \\ 0.1t &= \ln(4) \end{aligned}$$

$$\begin{aligned} t &= \ln(4)/0.1 \\ &\approx 13.863 \end{aligned}$$

3. If the continuous exponential growth rate is 8%/hour and you start with an amount of 300 units. How many units do you have after 5 hours?

$$y = Ae^{kt} \quad k = 8\%, \quad A = 300 \text{ units.}$$

$$y = 300e^{0.08t}$$

$$t = 5$$

$$y = 300e^{0.08 \cdot 5} = 300e^{0.4} \approx 447.547$$

447 UNITS

(OR 448 UNITS)

4. A student began taking the AP History test at 8am (
- $t = 0$
- hours). The student had 100% brain energy (
- $y$
- in percent) at that time. By 12noon, the student had 40% brain energy left. Use an exponential model to find the following.

- a. Find the half life (
- $T$
- in hours) of brain energy.

METHOD 2

$$y = Ae^{kt}$$

FIND  $k$ 

$$40 = 100e^{k \cdot 4}$$

$$k = \ln(0.4)/4 \approx -0.22907 \text{ per hour.}$$

$$\begin{aligned} \text{THEN FIND } T \\ 50 &= 100e^{-0.22907 \cdot T} \end{aligned}$$

$$\ln(y_2) = -0.22907 \cdot T$$

$$T = \frac{\ln(y_2)}{-0.22907} \approx 3.026$$

hours

≈ 3.0 hours

- b. Predict the brain energy left at 3pm.

$$y = 100e^{-0.22907 \cdot 7} \approx 20.119\%$$

$$\left\{ \text{OR } y \approx 100\left(\frac{1}{2}\right)^{(7/3.0)} \approx 20\% \right\}$$

≈ 20%