Name $\boxed{\text{KEY}}$

Algebra II

Sheet#1205 Review

Period ___

1. Solve by linear combination (elimination)

$$\begin{cases} 7x-12y=-22 & [1] \\ -5x+8y=14 & [2] \end{cases}$$

Alt: [1]*5, [2]*7

$[1]\times 2$: $14x-24y=-44$

$[2]\times 3$: $-15x+24y=42$

$-x = -2$

$x=2$. $-5(2)+8y=14$. $8y=24$

$y=3$

$\boxed{(2,3)}$

Solve by any method.

2. $-x+2y-17=0$

$+ \quad x-3y+28=0$

$-y+11=0$

$y=11$

$-x+22-17=0$

$x=5$

$\boxed{(5,11)}$ ✓

3. $2(-6m+3n=-24)$

$3(4m-2n=12)$

$-12m+6n=-48$

$12m-6n=36$

$0=12$

$\boxed{\emptyset}$ ✓

No solutions.

4. $y=2x-1$

$10x-5y=5$

$10x-5(2x-1)=5$

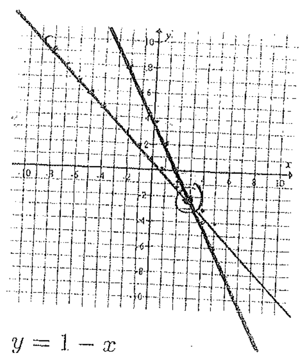
$10x-10x+5=5$

$5=5$

$\boxed{\text{Infinite \# solutions}}$ ✓

Solve by graphing.

5.



$y=1-x$

$2x+y=4$

$y=4-2x$

$\boxed{(3,-2)}$ ✓

Write as a single logarithm.

6. $2\log m+7\log n$

$\boxed{\log(m^2\cdot n^7)}$ ✓

7. Use the formula $\log_b M=\dfrac{\log_a M}{\log_a b}$ to change $\log_9 7.49$ to a base 10 log. Then find the value of the log to the nearest ten-thousandth.

$$\frac{\log(7.49)}{\log(9)}=\boxed{0.9164}$$ ✓

Solve.

8. $5=2e^{1+x}$

$\frac{5}{2}=e^{1+x}$

$\ln(5/2)=1+x$

$\boxed{x=\ln(5/2)-1\approx-0.0837}$ ✓

9. $\log_8(2x+1)=-1$

$8^{\log_8(2x+1)}=8^{-1}$

$2x+1=\frac{1}{8}$

$2x=\frac{1}{8}-1$

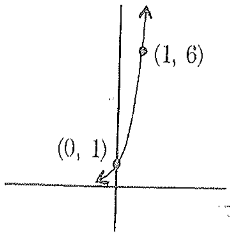
$2x=\frac{-7}{8}$

$\boxed{x=-7/16}$ ✓

Write the equation of the graph.

ASSUME EXPONENTIAL.

10.



① $y = A\cdot b^x$

$1 = A\cdot b^0$

$A = 1$

$y = 1\cdot b^x$

$6 = 1\cdot b^1$

$b = 6$

$\boxed{y = 6^x}$

② ALTERNATE: SAME RESULT.

$y = Ae^{kx}$

$1 = A\cdot e^0$

$A = 1$

$y = 1\cdot e^{kx}$

$6 = e^{k\cdot 1}$

$\ln(6) = k$

$\boxed{y = e^{\ln(6)x}}$

State the first 5 terms of the sequence whose nth term is given by a_n.

11. $a_n = -\frac{3}{n}$

$\boxed{-\frac{3}{1}, -\frac{3}{2}, -\frac{1}{1}, -\frac{3}{4}, -\frac{3}{5}}$

$a_1 = -\frac{3}{1} = -3$

$a_2 = -\frac{3}{2}$

$a_3 = -\frac{3}{3} = -1$

$a_4 = -\frac{3}{4}$

$a_5 = -3/5$

State the next 2 terms of the sequence and give a formula for the nth term.

12. 63, 54, 45, 36, 27 $, \boxed{18, 9}$

$d = -9$

$a_n = 63 + (n-1)(-9)$

$= 63 + 9n + 9$

$\boxed{a_n = 9n + 72}$

Find the sum.

13. $\sum_{n=1}^{4} n^4 =$

$a_1 = 1^4$

$a_2 = 2^4$

$a_3 = 3^4$

$a_4 = 4^4$

$= 1 + 16 + 81 + 256$

$= \boxed{354}$

Express using sigma notation.

14. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \frac{1}{243}$

$n = 6$

$r = -\frac{1}{3}$

$\boxed{\sum_{k=1}^{6} 1\cdot\left(-\frac{1}{3}\right)^{k-1}} = \sum_{k=1}^{6} -3\left(\frac{-1}{3}\right)^{k}$

15. State the 8th term of the geometric sequence $-9, \frac{9}{2}, -\frac{9}{4}, \ldots$

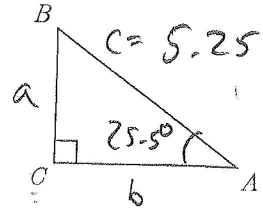
$r = -\frac{1}{2}$

$-9\left(-\frac{1}{2}\right)^{8-1} =$

$= -9\cdot\frac{1}{-128} = \boxed{9/128}$

-9
4.5
-2.25
1.125
-0.5625
0.28125
-0.140625
$0.0703125 = 9/128$

16. Solve the right triangle if $\angle A = 25.5°$ and $c = 5\frac{1}{4}$ feet. Give lengths to 3 significant figures and angles to the nearest tenth of a degree.



$B = 90° - 25.5°$

$= \boxed{64.5°}$

$\sin(25.5°) = \frac{a}{5.25}$

$a = 5.25\cdot\sin(25.5°) \approx \boxed{2.26\text{ ft}}$

$b = 5.25\cos(25.5°) \approx \boxed{4.74\text{ ft}}$

[Check: $\sqrt{2.26^2 + 4.74^2} = 5.25$ ft ✓]

17. Solve the right triangle shown in the previous problem if $a = 11$ millimeters and $b = 21$ millimeters. Give lengths to 3 significant figures and angles to the nearest tenth of a degree.

$\tan(A) = \frac{11}{21}$

$A = \boxed{\tan^{-1}}(11/21) = \boxed{27.6°}$

$B = 90° - 27.6° = \boxed{62.4°}$

$c = \sqrt{11^2 + 21^2} = \sqrt{121 + 441} = \sqrt{562}$

$\approx \boxed{23.7\text{ mm}}$

18. A moving van from Wally's Rentals costs \$45 per day and 10¢ per mile. The same van from Rent-A-Truck costs \$70 per day and 6¢ per mile. For what number of miles is the daily cost of the two vans the same?

$y = 45 + 0.1\cdot x$

y = cost
x = # miles

$y = 70 + 0.06x$

$45 + 0.1x = 70 + 0.06x$

$0.04x = 25$

$x = \boxed{625\text{ MILES}}$

19. Charlotte's coin box contains only pennies and nickels. If she has 66 coins worth \$2.38, how many of each type of coin does she have?

x = # pennies
y = # nickels

NUMBER: $x + y = 66$

$(1\cdot x + 5\cdot y = 238)$ (cents)

$x + 5y = 238$

$-x - y = 66$

$4y = 172$

$y = 43$

$x = 66 - 43 = 23$

[Check: $43 + 23 = 66$ ✓
$23 + 5\cdot 43 = 23 + 215 = 238$ ✓]

$\boxed{\text{23 pennies, 43 nickels}}$

20. José owns a rare baseball card which increases in value at a rate of 15% per year. If the card was worth \$3.50 in 1990, how much is it worth in 1995?

COMPOUNDED	CONTINUOUS (NOT LIKELY INTERPRETATION OF QUESTION)
$y = A(1+\frac{i}{m})^{mt}$	$y = Ae^{kt}$
$m = 1/\text{year}$, $i = 0.15$, $A = 3.50$	$y = 3.50 \cdot e^{0.15 \cdot t}$, $t = 0$ is 1990
$y = 3.50(1.15)^5 = 3.50 \cdot 2.0114 = \boxed{\$7.04}$	$y(t=5) = 3.50 \cdot e^{0.15 \cdot 5}$, $y = 3.50 \cdot 2.117 = \boxed{\$7.41}$

21. A certain satellite has a power supply whose output in watts is given by the equation $P = 40e^{\frac{-t}{900}}$, where t is the number of days the battery has operated.

a) If it is operated continuously after the satellite is placed into orbit, how many watts is the battery putting out after one year?

b) If it takes at least 10 watts to operate the satellite, how many days can the satellite be used?

a, $y(t=365) = 40 \cdot e^{-365/900} \approx 40 \cdot 0.6666$
$\approx \boxed{26.66 \text{ watts}}$

b, $10 \geq 40e^{-t/900}$

Solve for t in equality

$\frac{1}{4} = e^{-t/900}$

$\ln(1/4) = -t/900$

$t = -900\ln(1/4)$
$= 900\ln(4) =$
$\approx 900 \cdot 1.3863 = \boxed{1247.7 \text{ days}}$

22. A leaf mite has infected palm trees and is killing them at a rate of 10% per year.

a) A plantation had 10,000 palm trees when the trees were originally infected. Find a formula relating the time the plantation has been infected to the number of trees remaining.

b) How many trees are left after 2 years? after 5 years? after 20 years?

c) When will less than 100 trees remain?

(1) COMPOUNDED (ANSWER KEY)

a) $y = 10000(1-0.1)^t$
$y = \boxed{10000(0.9)^t}$

b, $y(2) = \boxed{8100 \text{ trees}}$
$y(5) = \boxed{5905}$, $y(20) = \boxed{1216}$

c, $100 = 10000(0.9)^t$
$\frac{1}{100} = 0.9^t$
$\ln(1/100) = \ln(0.9^t)$
$\ln(1/100) = t\ln(0.9)$
$t = \ln(1/100)/\ln(0.9) \approx \boxed{43.7 \text{ yrs}}$

(2) CONTINUOUS (MY PREFERENCE)

a/ $y = 10000e^{-0.10t}$

b, $y(2) = 8187$
$y(5) = 6065$, $y(20) = 1353$.

c, $\ln(1/100) = -0.10t$
$t = -\ln(1/100)/0.10 = 46.1 \text{ yrs}$

23. The half-life of a radioactive material can be found by using the formula $N = N_0e^{-kt}$, where k is a constant unique to the material and t is measured in years. Find the half-life of Plutonium-238 given that its constant k is 0.008022.

CONTINUOUS

$\frac{1}{2} = e^{-0.008022 \cdot t}$

$\ln(1/2) = -0.008022t$

$t = \ln(1/2)/(-0.008022) = \boxed{86.4 \text{ yrs}}$

24. The total daily profit P, in dollars on the production of x items is $P(x) = -2x^2 + 64x - 12$. How many items must be produced each day to maximize profit? What is the maximum profit?

TI-83 GRAPH (UNLESS YOU WANT TO USE CALCULUS!)

WINDOW $\underbrace{0, 30, 10}_{x}$, $\underbrace{0, 600, 100, 1}_{y}$

MAX (16, 500) $\boxed{\text{16 items, \$500 profit}}$

25. The first 3 terms of an arithmetic progression are -15, -17, and -19. Find the 22nd term and the sum of the first 22 terms.

$d = -2$

$a_n = -15 + (n-1)\cdot(-2)$
$a_n = -15 - 2n + 2$
$a_n = -2n - 13$
$\boxed{a_{22} = -57}$

$S_{22} = 22\left(\frac{-15-57}{2}\right)$
$\boxed{S_{22} = -792}$

26. How many terms of the arithmetic series $(-15) + (-8) + (-1) + \cdots$ must be added to give a sum of 801?

$d = +7$

$a_n = -15 + (n-1)\cdot 7$
$a_n = 7n - 22$

$S_n = n\left(\frac{-15 + a_n}{2}\right)$

$801 = n\left(\frac{-15 + 7n - 22}{2}\right)$
$801 = \frac{n}{2}(7n - 37)$
$801 = \frac{7}{2}n^2 - \frac{37}{2}n$
QUADRATIC EQ. *
$\boxed{n = 18}$

$\frac{7}{2}n^2 - \frac{37}{2}n - 801 = 0$

$x = \frac{\frac{37}{2} + \sqrt{(\frac{37}{2})^2 + 4(\frac{7}{2})801}}{2(7/2)}$

$= \frac{126}{7} = 18.$

27. Find the sum of the first 6 terms of the geometric series $80 + (-20) + 5 + \cdots$.

$S_6 = 80\left(\frac{1-(-1/4)^6}{1-(-1/4)}\right) = 80\frac{(4095/4096)}{(5/4)}$

$= 4095/64 = \boxed{63\frac{63}{64}}$

28. $\frac{1}{2} + (-\frac{1}{4}) + \frac{1}{8} + \cdots$

INFINITE $r = -1/2$

$S = \frac{1}{2}\left(\frac{1}{1-(-1/2)}\right) = \frac{1}{2}\left(\frac{1}{3/2}\right) = \boxed{\frac{1}{3}}$

29. A log pile has 61 logs in the bottom layer, 58 in the second layer, 55 in the third layer, and so on. If there are 639 logs in the pile, how many layers are there?

METHOD: SEE QUESTION 26. $d = -3$

$a_n = 61 + (n-1)\cdot(-3) = -3n + 64$

$639 = n\left(\frac{61 + -3n + 64}{2}\right)$

$-\frac{3}{2}n^2 - \frac{125}{2}n + 639 = 0$

QUADRATIC EQ.
$3n^2 - 125n + 1278 = 0$
Solutions: 18, $23\frac{2}{3}$.
Pick $n = 18$

$\boxed{n = 18}$ ANSWER IS A COINCIDENCE WITH QUESTION 26.

$\rightarrow 1000(14+14.5+15+15.5+16+16.5+17+17.5+18+18.5)$ 5 years
$= 162.5 \cdot 1000 = 162{,}500$

30. Two employees, Ms. Jones and Mr. Martin, are hired by a company at the same time, each at a starting salary of \$28,000. Ms. Jones agrees to an annual raise of \$2000 while Mr. Martin agrees to a semi-annual increase of \$500. After 5 years have elapsed, who will have earned more money? How much more?

JONES
1 28000
2 30000
3 32000
4 34000
5 36000
160000

$n \Rightarrow$ years
$a_n = 28000 + 2000(n-1)$
$S_n = 5\left(\frac{28000+36000}{2}\right)$
$= 160{,}000$

MARTIN $m \Rightarrow 1/2$ yrs.
$a_m = 14000 + 500(m-1)$
$S_m = 10\left(\frac{14000+18500}{2}\right)$
$= 162{,}500.$
$\boxed{\$2500 \text{ MORE FOR Mr. Martin}}$

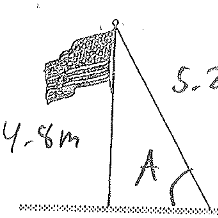
DISAGREES with ANSWER KEY

times

31. After the first swing, the path of a pendulum bob is 0.9 ~~as long~~ as long as the previous swing. If the first swing is 50 cm long, how far does the bob travel on the sixth swing? What total distance does the bob travel in those six swings?

$a_n = 50(0.9)^{n-1}$
$a_6 = 50(0.9)^5 \approx \boxed{29.5\text{cm}}$
$S_6 = 50\left(\frac{1-(0.9)^6}{1-0.9}\right) =$
$= 50\left(\frac{1-0.9^6}{0.1}\right) = 500(1-0.9^6) =$
$\approx \boxed{234.3\text{ cm}}$

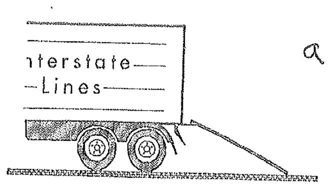
32. A wire 5.2 meters long is attached to the top of a flagpole 4.8 meters long. Approximately what is the measure of the angle the wire makes with the ground? Round your answer to the nearest tenth of a degree or nearest ten minutes.



5.2m
4.8m
A

$\sin(A) = \frac{4.8}{5.2}$
$A = \boxed{\sin^{-1}}(4.8/5.2)$
$\approx \boxed{67.4°}$

33. A 1.3 meter long ramp makes an angle of 29° with the ground. Approximately how far above the ground is the back end of the ramp where it meets the truck? Round your answer to the nearest tenth.



1.3m
a
29°

$\sin(29°) = \frac{a}{1.3}$
$a = 1.3\sin(29°)$
$= \boxed{0.63\text{ m}}$

34. **Formulas**

- Arithmetic sequences and series:
$a_n = a_1 + (n-1)d$ and $S_n = n\left(\frac{a_1+a_n}{2}\right)$.
- Geometric sequences and series:
$a_n = a_1(r)^{n-1}$ and $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$, and for $n \to \infty$ with $|r| < 1$, $S = a_1\left(\frac{1}{1-r}\right)$.
- Compounded interest:
$y = A\left(1+\frac{i}{m}\right)^{mt}$, where i = annual interest, t = number of years, and m = number of compounding periods per year. See also geometric sequences.
- Continuous growth and decay:
$y = Ae^{kt}$, where k = growth or decay rate per unit of time and t = elapsed time.
- Trigonometry:
$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$, $\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$, and $\tan(\theta) = \frac{\text{opp}}{\text{adj}}$.

Answer List

1. (2, 3)
2. (5, 11)
3. $\varnothing$
4. coincide
5. (3, −2)
6. $\log m^2n^7$
7. $\frac{\log 7.49}{\log 9} = 0.9164$
8. $x = -1 + \ln 2.5 \approx -0.08$
9. $x = \frac{-7}{16}$
10. $y = 6^x$
11. $-3, -\frac{3}{2}, -1, -\frac{3}{4}, -\frac{3}{5}$
12. 18, 9; $a_n = 72 - 9n$
13. 354
14. $\sum_1^6 (-\frac{1}{3})^{k-1}$
15. $\frac{9}{128}$
16. $\angle B = 64.5°$, $a \approx 2.26$ ft, $b \approx 4.74$ ft
17. $\angle A \approx 27.6°$, $\angle B \approx 62.4°$, $c = \sqrt{562} \approx 23.7$ mm
18. 625
19. 23 pennies, 43 nickels
20. \$7.04
21. 26.66 watts; 1247.7 days
22. $N(t) = 10{,}000(0.9)^t$; 8100, 5905, 1216; 43 years 43.7 years
23. 86.4
24. 16 items; \$500
25. −57, −792
26. 18
27. $63\frac{63}{64}$
28. $\frac{1}{3}$
29. 18
30. ~~Ms. Jones~~: \$2500 Mr. Martin
31. ≈ 29.5 cm; ≈ 234.3 cm
32. 67.4° or 67°20′
33. 0.6 m